1\. a) $\bar{x} = 24.7$

$\tilde{x} = 24.5$

$s^2 = 111.1$

$s = 10.5$

$R = 46$

b) $\bar{x}_{tr(10)} = 24.9$ → The trimmed values indicate that this data set is centred and not affected by its outliers.

$\tilde{x}_{tr(10)} = 24.5$

$S^2_{tr(10)} = 55.8$

$S_{tr(10)} = 7.5$

$R_{tr(10)} = 27$

c)

Stem	leaf	frequency
0	345577	6
1	1222345668899	12
2	01112223333444455556677779999	28
3	222223355577888899	17
4	00239	5

Stem-and-Leaf Plot

Stem	Leaf	frequency
0	34	2
0*	5577	4
1	122234	6
1*	5668899	6
2	011122233334444	15
2*	5555667777999	13
3	2222233	7
3*	5557788889 9	10
4	0023	4
4*	9	1

Double Stem-and-Leaf Plot

The plots seem to be highly symmetrical.

d) See end for histogram.

2 a) There are $2^5 = \boxed{32}$ outcomes

b) $W = \{(1,1,0,0,0), (1,1,1,0,0), (1,1,0,1,0), (1,1,1,1,0),$
$(1,1,0,0,1), (1,1,1,0,1), (1,1,0,1,1), (1,1,1,1,1),$
$(0,0,1,1,0), (1,0,1,1,0), (0,1,1,1,0), (0,0,1,1,1),$
$(1,0,1,1,1), (0,1,1,1,1), (1,0,1,0,1)\}$

c) A contains $2^3 = \boxed{8}$ outcomes

d) $A \cap W = \{(1,1,0,0,0), (1,1,1,0,0)\}$

3. a) $E \cap F' \cap G'$

b) $E \cap F' \cap G$

c) $E \cup F \cup G$

d) $(E \cap F) \cup (E \cap G) \cup (F \cap G)$

e) $E \cap F \cap G$

f) $E' \cap F' \cap G'$

g) $((E \cap F) \cup (E \cap G) \cup (F \cap G))'$

h) $(E \cap F \cap G)'$

i) $(E' \cap F \cap G) \cup (E \cap F' \cap G) \cup (E \cap F \cap G')$

j) $E \cup E'$

4. $(A \cup B) \cap (A' \cup B) \cap (A \cap B') = \boxed{A \cap B}$

5. a) The chance that she will open the door on any specific try is equal,

$$P = \frac{\cancel{3}}{4} \cdot \frac{\cancel{2}}{\cancel{3}} \cdot \frac{1}{\cancel{2}} = \frac{1}{4} = \boxed{0.25}$$

b) $$P = \frac{3}{4} \cdot \frac{3}{4} \cdot \frac{1}{4} = \frac{9}{64} = \boxed{0.14}$$

6. $$P = \frac{\cancel{5}}{\cancel{15}} \cdot \frac{\cancel{4}}{\cancel{14}\,7} \cdot \frac{10}{13} \cdot \frac{\cancel{9}}{\cancel{12}} \cdot \frac{\overset{4}{\cancel{8}}}{11} = \frac{40}{1001} = \boxed{0.04}$$

7. $P(A) = P(B) = P(C) = 0.5$

a) $P(C|B) = \begin{cases} 0 & B \neq A \\ 1 & B = A \end{cases} \neq P(C)$

$\therefore$ $\{A, B, C\}$ is not an independent set

b) $P(B|A) = 0.5 = P(B)$

$P(C|A) = 0.5 = P(C)$

$\therefore$ $\{A, B\}$ and $\{A, C\}$ make up independent sets

8. Let J_i be the event where Judge i votes guity and G be the event where the defendant is guilty

given, $P(G) = 0.8$

$P(J_i | G) = 0.7$

$P(J_i | G') = 0.2$

The events $\{J_3, J_2, J_1\}$ are independent

a) finding $P(J_3 | J_2 \cap J_1)$,

$$P(J_3 | J_2 \cap J_1) = \frac{P(J_3 \cap J_2 \cap J_1)}{P(J_2 \cap J_1)}$$

now since J_3, J_2, J_1 are indepent of each other but not of G,

$$\frac{P(J_3 \cap J_2 \cap J_1)}{P(J_2 \cap J_1)} = \frac{P(G)P(J_1|G)P(J_2|G)P(J_3|G) + P(G')P(J_1|G')P(J_2|G')P(J_3|G')}{P(G)P(J_1|G)P(J_2|G) + P(G')P(J_1|G')P(J_2|G')}$$

$$= \frac{0.8 \cdot 0.7^3 + 0.2 \cdot 0.2^3}{0.8 \cdot 0.7^2 + 0.2 \cdot 0.2^3} = \frac{0.276}{0.4} = \boxed{0.69}$$

c) finding $P(J_3 | J_2' \cap J_1')$,

$$P(J_3 | J_2' \cap J_1') = \frac{P(J_3 \cap J_2' \cap J_1')}{P(J_2' \cap J_1)}$$

$$= \frac{P(G)P(J_1'|G)^2 P(J_3|G) + P(G')P(J_1'|G')^2 P(J_3|G')}{P(G)P(J_1|G)^2 + P(G')P(J_1'|G')^2}$$

$$= \frac{0.8 \cdot 0.3^2 \cdot 0.7 + 0.2 \cdot 0.8^2 \cdot 0.2}{0.8 \cdot 0.3^2 + 0.2 \cdot 0.8^2} = \frac{0.076}{0.2} = \boxed{0.38}$$

b) finding $P(J_3 | J_2' \cap J_1) = P(J_3 | J_2 \cap J_1')$,

$$P(J_3 | J_2' \cap J_1) = \frac{P(J_3 \cap J_2' \cap J_1)}{P(J_2' \cap J_1)}$$

$$= \frac{P(G)P(J_1|G)^2 P(J_2'|G) + P(G')P(J_1|G')^2 P(J_2'|G')}{P(G)P(J_1|G)P(J_2'|G) + P(G')P(J_1|G')P(J_2'|G')}$$

$$= \frac{0.8 \cdot 0.7^2 \cdot 0.3 + 0.2 \cdot 0.2^2 \cdot 0.8}{0.8 \cdot 0.7 \cdot 0.3 + 0.2 \cdot 0.2 \cdot 0.8} = \frac{0.124}{0.2} = \boxed{0.62}$$

9.

	x	2	3	4	5	6	7	8	9	10	11	12
a)	$f(x)$	$\frac{1}{36}$	$\frac{1}{18}$	$\frac{1}{12}$	$\frac{1}{9}$	$\frac{5}{36}$	$\frac{1}{6}$	$\frac{5}{36}$	$\frac{1}{9}$	$\frac{1}{12}$	$\frac{17}{18}$	$\frac{1}{36}$
b)	$F(x)$	$\frac{1}{36}$	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{5}{18}$	$\frac{5}{12}$	$\frac{7}{12}$	$\frac{13}{18}$	$\frac{5}{6}$	$\frac{11}{12}$	$\frac{35}{36}$	1

c) $P(2 \leq X < 7) = F(6) - 0 = \boxed{\frac{5}{12}}$

10. a) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_{3.8}^{4} a(x-3.8)\,dx - \int_{4}^{4.2} a(x-4.2)\,dx = a\int_{3.8}^{4}(x-3.8)\,dx + \int_{4}^{4.2}(4.2-x)\,dx$$

$$= a\left[\frac{x^2}{2} - 3.8x\right]_{3.8}^{4} + a\left[4.2x - \frac{x^2}{2}\right]_{4}^{4.2}$$

$$= a\left(\frac{4^2}{2} - 3.8\cdot 4 - \frac{3.8^2}{2} + 3.8^2 + 4.2^2 - \frac{4.2^2}{2} - 4.2\cdot 4 + \frac{4^2}{2}\right)$$

$$= a\left(4^2 + \frac{3.8^2}{2} + \frac{4.2^2}{2} - 4(\cancel{3.8+4.2}\ 8)\right)$$

$$= 0.04a = 1$$

$\therefore \quad a = \frac{1}{0.04} = \boxed{25}$

b) $P(3.9 < x < 4.3) = \int_{3.9}^{4.3} f(x)\,dx = 1 - \int_{3.8}^{3.9} f(x)\,dx = 1 - \int_{3.8}^{3.9} 25(x-3.8)\,dx$

$$= 1 + 25\left[3.8x - \frac{x^2}{2}\right]_{3.8}^{3.9} = 1 + 25\left(3.8\cdot 3.9 - \frac{3.9^2}{2} - 3.8^2 + \frac{3.8^2}{2}\right)$$

$$= \boxed{0.875}$$

c)
$$F(x) = \int_{-\infty}^{x} f(x)\,dx = \begin{cases} \int_{3.8}^{x} 25(x-3.8)\,dx & 3.8 \leq x < 4 \\ \int_{3.8}^{4} 25(x-3.8) - \int_{4}^{x} 25(x-4.2) & 4 < x \leq 4.2 \\ 0 & x < 3.8 \\ 1 & x > 4.2 \end{cases}$$

for $3.8 \leq x < 4$,

$$F(x) = 25\left[\frac{x^2}{2} - 3.8x\right]_{3.8}^{x} = 25\left(\frac{x^2}{2} - 3.8x - \frac{3.8^2}{2} + 3.8^2\right) = 12.5x^2 - 95x + 180.5$$

for $4 < x \leq 4.2$

$$F(x) = 25\left[4.2x - \frac{x^2}{2}\right]_{4}^{x} + 12.5\cdot 4^2 - 95\cdot 4 + 180.5 = -12.5x^2 + 105x - 220 + 0.5$$

$$\therefore \boxed{F(x) = \begin{cases} 1 & x \geq 4.2 \\ -12.5x^2 + 105x - 199.5 & 4 \leq x < 4.2 \\ 12.5x^2 - 95x + 180.5 & 3.8 < x < 4 \\ 0 & x \leq 3.8 \end{cases}}$$

graph at end.

11. a)

x	-1	0	1
$g(x)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

b)

y	-1	0	1
$h(y)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

c) $P(X>Y) = f(1,0) + f(0,-1)$

$= \boxed{\frac{1}{2}}$

d) $P(X \geq 0 \mid Y \leq 0) = \dfrac{f(-1,0)+f(0,-1)}{h(-1)+h(0)} = \dfrac{\frac{1}{2}}{\frac{3}{4}} = \boxed{\frac{2}{3}}$

e) $g(1)h(0) = \frac{1}{4} \cdot \frac{1}{2} = \frac{1}{8} \neq f(1,0)$

$\therefore$ X and Y are <u>not</u> independent

12. a) $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,dx\,dy = 1$

$\therefore \int_0^1\int_0^2 a(1+xy)\,dx\,dy = 1$

$a\int_0^1\int_0^2 (1+xy)\,dx\,dy = a\int_0^1 \left[x + \frac{x^2y}{2}\right]_0^2 dy = a\int_0^1 (2+2y)\,dy = a(2+1) = 3a.$

$\therefore a = \boxed{\frac{1}{3}}$

b) $f(x|y) = \dfrac{f(x,y)}{h(y)} = \dfrac{\cancel{\frac{1}{3}}(1+xy)}{\cancel{\frac{1}{3}}\int_0^2 (1+xy)\,dx} = \boxed{\dfrac{1+xy}{2+2y}}$

c) $f(y|x) = \dfrac{1+xy}{\int_0^1 (1+xy)\,dy} = \dfrac{1+xy}{1+\frac{x}{2}}$

$\therefore P(\frac{1}{2} < Y < 1 \mid X=1) = \int_{\frac{1}{2}}^{1} \frac{1+y}{\frac{3}{2}}\,dy = \frac{2}{3}y + \frac{1}{3}y^2 \Big|_{\frac{1}{2}}^{1} = 1 - \frac{1}{3} - \frac{1}{12} = \boxed{\frac{7}{12}}$

1. d)

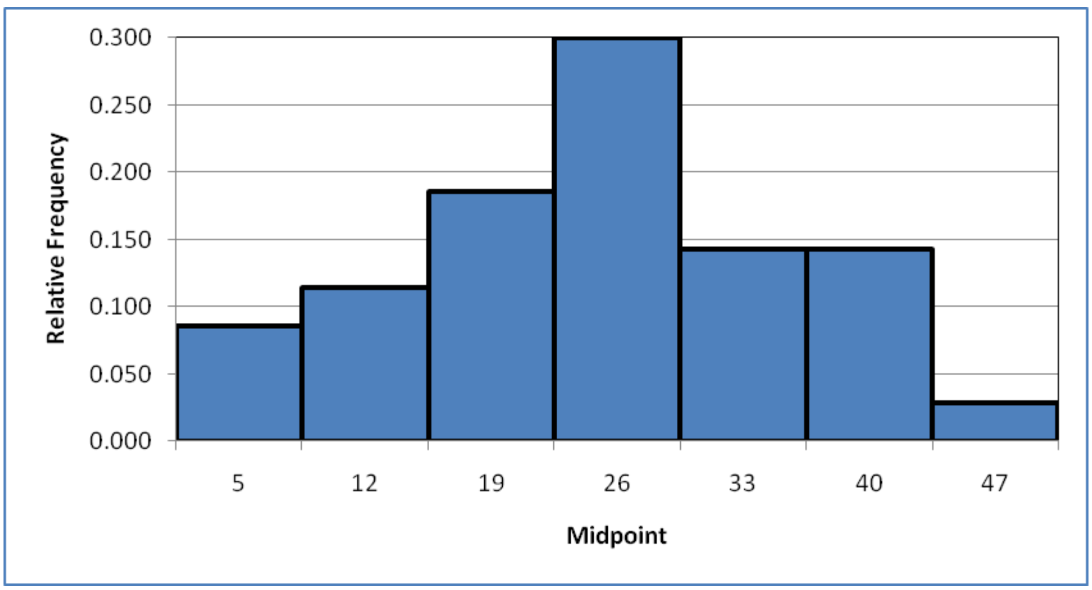
Using Sturges' Rule to find the number of bins,

$\lfloor 3.322 \log 70 + 1 \rfloor = \lfloor 6.13 + 1 \rfloor = 7$

Since the range is 46, we add 3 to it to make it divisible by 7; instead of the values ranging from 3 to 49, they now range from 1.5 to 50.5,

Bin	Range	Midpoint	Frequency	Relative Freq.
1	[1.5,8.5)	5	6	0.086
2	[8.5,15.5)	12	8	0.114
3	[15.5,22.5)	19	13	0.186
4	[22.5,29.5)	26	21	0.300
5	[29.5,36.5)	33	10	0.143
6	[36.5,43.5)	40	10	0.143
7	[43.5,50.5]	47	2	0.029

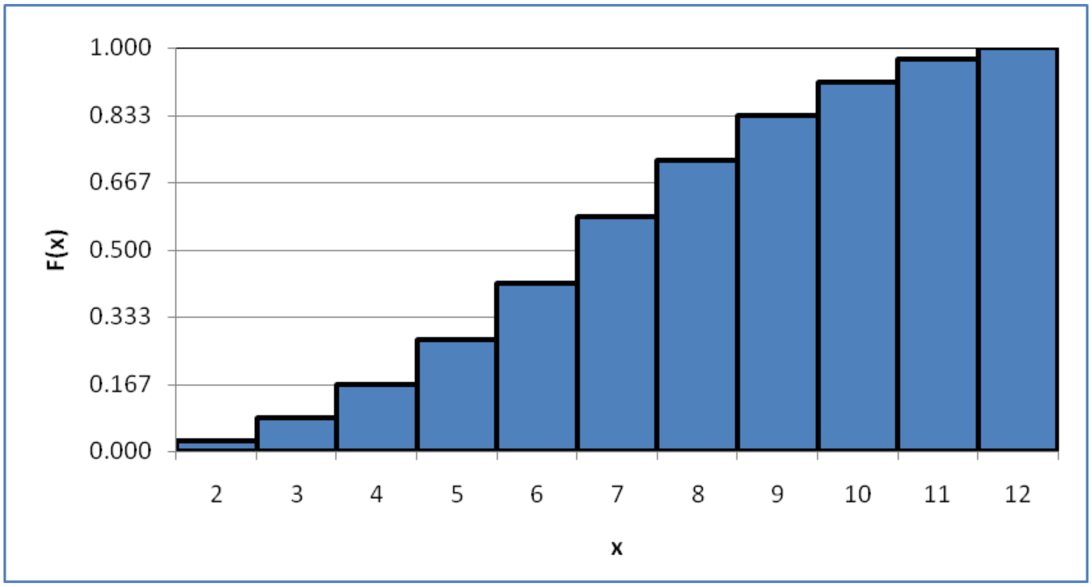
Plotting the above data,



The distribution seems to come from a bell shaped distribution that is right-skewed as there are more values to the left of Bin 4 than to its right.

9. bcont.)

Graph of F(x):



10. dcont.)

Graph of F(x):