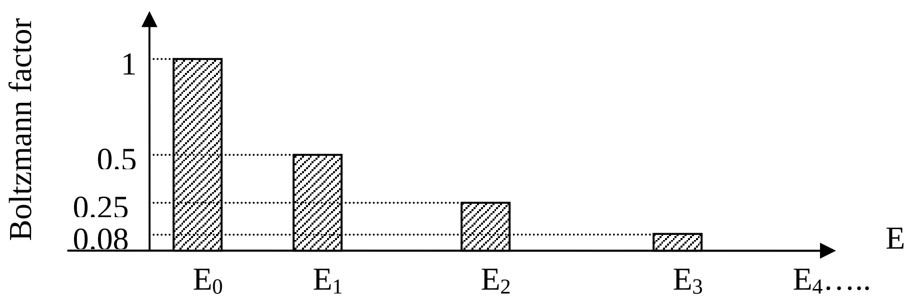
PHY 292S **Midterm test** **Winter 2007**

Friday, January 26th, 9:05am - 9:55am

No calculators are allowed. Necessary formulae may be found in the attached formulae sheet.

1. The following bar graph shows the Boltzmann factors of the states of a hypothetical system at one particular temperature. The horizontal axis is energy, and the energy is measured from the ground state, that is, E_0=0. If the partition function at this temperature is 2.0, what is the probability of this system to be in the first excited state (E=E_1) at this temperature? (10 points)



2. In the class, we have considered the Einstein solid model in the limit of high temperature (q >> N >> 1). In this problem, we will work out the Einstein solid model in the limit of low temperature (1<< q << N). Note that we will also use the convention that the energy U of the system is given by U=qε, where ε is an energy unit.

   (a) With the use of Stirling's approximation, one can show that the multiplicity of an Einstein solid in the low-temperature limit is given by $\Omega = \left({eN}/{q} \right)^q$. Using this, find a formula for the entropy of an Einstein solid in the low temperature limit. (10 points)

   (b) Using the definition of temperature, and using the entropy you found in part (a), show that the energy as a function of temperature is given by

   $$U = \varepsilon N e^{-\varepsilon/kT}$$. (10 points)

   (c) Find the heat capacity of an Einstein solid in the low-temperature limit. You should find heat capacity at constant volume using the results given in part (b). (10 points)

   (d) What is the behaviour of C_v in the limit of $T \to 0$? (5 points)

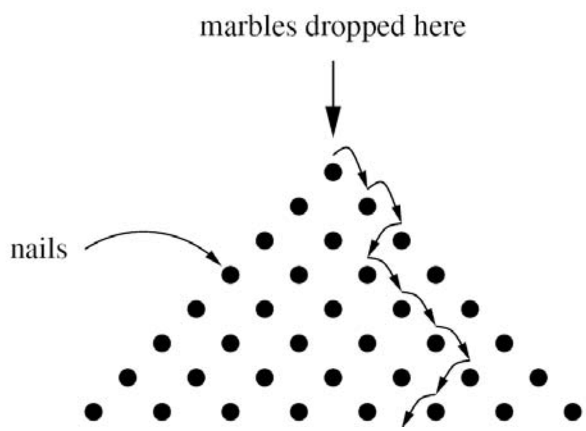
   (e) What is the heat capacity of an Einstein solid predicted by the Equipartition theorem? Note that an Einstein solid is a collection of N oscillators with only vibrational degrees of freedom. (5 points)

   (f) Compare the results in part (d) and (e). If they are different, why? (5 points)

(continued on the next page)

3. Recall that we studied a three-dimensional ideal gas in the class. Now let us consider an ideal monatomic gas that lives in a two-dimensional universe, occupying an area A instead of volume V. Following the same logic as in the three dimensional case, you will find a formula for the multiplicity of this gas, which has N molecules.
   (a) What is the dependence of multiplicity Ω on area, A? (5 points)
   (b) What is the dependence of multiplicity Ω on energy, U? You will have to consider the momentum space volume as discussed in the class. (10 points)
   (c) What is the total multiplicity? You can assume that you can write the N dependence of the multiplicity which does not explicitly depend on A or U as f(N). (5 points)
   (d) What is the entropy of this two-dimensional gas? (5 points)

4. A famous carnival toy is a machine that drops a large number of marbles at the top of a pyramid of nails tacked to a board like this:



The marbles fall down and at each step have a 50% chance of going right or left.

   (a) If there are N rows of nails total, write down an expression for the number of paths $\Omega(N,r,l)$ that a marble can take that make a total of l steps to the left and r steps to the right. (10 points)
   (b) If N = 8, how many ways are there of traveling distance x = 8 to the right? Here the distance x is defined as $x = r - l$, and is measured horizontally from where the marbles start, i.e., from a line down the middle of the picture above. (5 points)
   (c) How many ways are there of traveling distance zero if N=8 ? (5 points)