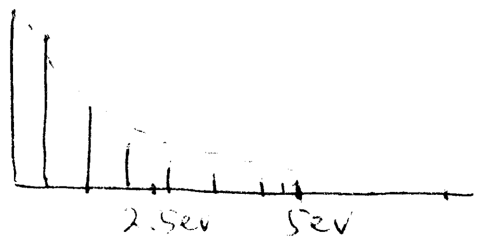
① System A $\quad 1\,eV \gg 300K = \frac{1}{40}\,eV$

System B $\quad 1\,meV \ll \frac{1}{400}\,eV = 2.5\,meV$

There are more energy levels within kT for System B.

System A



System B

$\boxed{\text{System B}}$ has larger Z.

② ratio is given by the Boltzmann factors

$$\frac{e^{-E_n/kT}}{e^{-E_p/kT}} = e^{-(E_n-E_p)/kT} = e^{-(m_n-m_p)c^2/kT}$$

$$= \exp\left(\frac{-2.3\times10^{-30}\times(3\times10^{8})^2}{1.38\times10^{-23}\times1.5\times10^{11}}\right)$$

$$= \exp\left(-\frac{10^{-13}}{10^{-12}}\right) = \exp(-0.1) \cong 0.9$$

③ Total radiation reaching Mars: $600\cdot\pi R^2$

" emitted by Mars: $\sigma T^4\cdot 4\pi R^2$

R: Mars's radius.

$$\therefore\ 600\cdot\cancel{\pi R^2} = \sigma T^4\cdot 4\cancel{\pi R^2}$$

$$\therefore\ T = \left(\frac{600}{4\sigma}\right)^{1/4} \cong 227\ K$$

④ Classical statistics can be used when $\frac{V}{N}$ is much larger than quantum volume v_Q

$$\frac{V}{N} \gg v_Q = \left(\frac{h^2}{2\pi m k T}\right)^{3/2}.$$

Therefore

$$T \gg \left(\frac{N}{V}\right)^{2/3} \frac{h^2}{2\pi m k}$$

$$N = 10^{23},\quad V = 10^{-6}\,m^3,\quad m = 4 \times 1.67\times10^{-27}\,kg$$

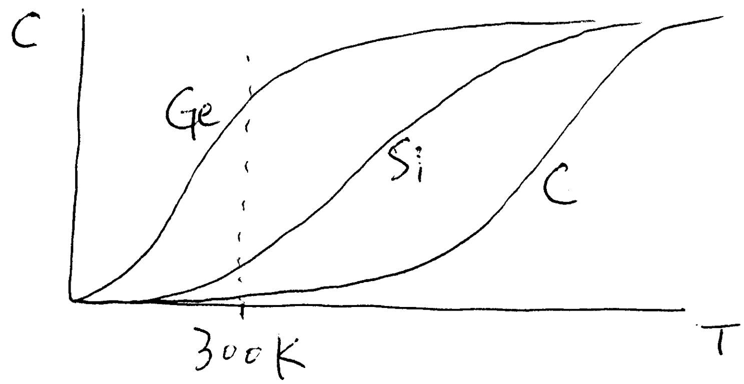
↑ He atom (full mark was given for missing this)

$$\left(\frac{10^{23}}{10^{-6}}\right)^{2/3} \cdot \frac{(6.6\times10^{-34})^2}{2\pi \cdot 4 \cdot 1.67\times10^{-27} \cdot 1.38\times10^{-23}}$$

$$= 2.15\times10^{19} \cdot 7.5\times10^{-19} \approx 16\ (K)$$

$$\therefore \quad \underline{\underline{T \gg 16\ K}}$$

⑤ Debye temperature is when the system reaches high-T (equipartition limit). Based on Debye temperature, one can expect the heat capacity should show the following T-dependence



Ge

⑥ (a) $Z = \sum_J (2J+1)\, e^{-J(J+1)\epsilon/kT}$

(b) $Z = 1 + 3e^{-2\epsilon/kT} + \cdots$

(c) $\bar{E} = -\frac{1}{Z}\frac{\partial Z}{\partial \beta} = -\frac{1}{Z}\frac{\partial}{\partial \beta}\left(1+3e^{-2\epsilon\beta}\right)$

$= -\frac{1}{1+3e^{-2\epsilon\beta}}\left(-6\epsilon e^{-2\epsilon\beta}\right) = \frac{6\epsilon e^{-2\epsilon\beta}}{1+3e^{-2\epsilon\beta}}$

(d) When $kT \ll \epsilon$, $\beta\epsilon \gg 1$

$\bar{E} \approx 6\epsilon e^{-2\epsilon\beta}$

$C_v = \frac{d\bar{E}}{dT} = 6\epsilon e^{-2\epsilon/kT}\left(\frac{+2\epsilon}{kT^2}\right) = 12k\left(\frac{\epsilon}{kT}\right)^2 e^{-2\epsilon/kT}$

(e) $Z = \int_0^\infty dJ\,(2J+1)\, e^{-J(J+1)\epsilon/kT}$

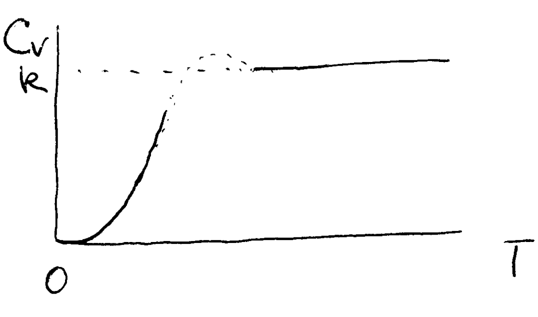
$x \equiv \frac{J(J+1)\epsilon}{kT}$ $\quad dx = (2J+1)\,dJ \cdot \frac{\epsilon}{kT}$

$\therefore Z = \int_0^\infty \frac{kT}{\epsilon}\, dx\, e^{-x} = \frac{kT}{\epsilon}\left[-e^{-x}\right]_0^\infty = \frac{kT}{\epsilon} = \frac{1}{\epsilon\beta}$

(f) $\bar{E} = -\frac{1}{Z}\frac{\partial Z}{\partial \beta} = -\epsilon\beta\left(-\frac{1}{\epsilon\beta^2}\right) = \frac{1}{\beta} = kT$

(g) $C_v = \frac{d\bar{E}}{dT} = k$

(h)



⑦ (a) $Z_1 = \sum_s e^{-E(s)/kT} = e^{-\mu B/kT} + 1 + e^{+\mu B/kT} = 1 + 2\cosh\left(\frac{\mu B}{kT}\right)$

For N moments:

$$Z = (Z_1)^N = \left[1 + 2\cosh\left(\frac{\mu B}{kT}\right)\right]^N$$

(b) $F = -kT \ln Z = -NkT \ln\left[1 + 2\cosh\left(\frac{\mu B}{kT}\right)\right]$

(c) $\frac{\bar{M}}{N} = \frac{1}{Z_1} \sum_s m(s)\, e^{-E(s)/kT}$

$$= \left(+\mu e^{+\mu B/kT} + 0 - \mu e^{-\mu B/kT}\right) / Z_1$$

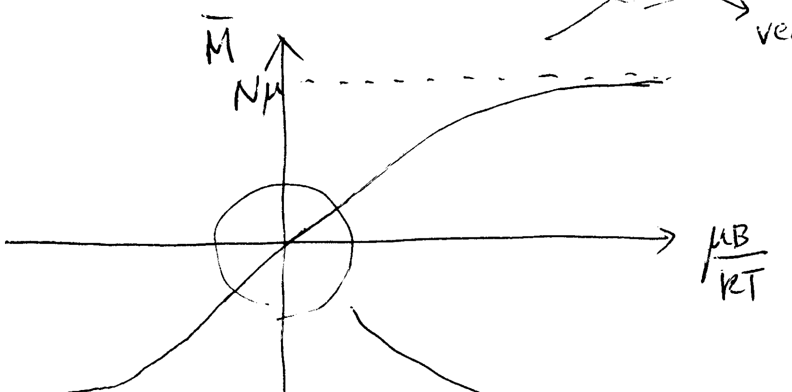
$$= \frac{\mu}{Z_1} 2\sinh\left(\frac{\mu B}{kT}\right)$$

$$\therefore \bar{M} = \boxed{\frac{2N\mu \sinh\left(\frac{\mu B}{kT}\right)}{1 + 2\cosh\left(\frac{\mu B}{kT}\right)}}$$

(d) $\frac{\mu B}{kT} \to 0$: $\bar{M} \Rightarrow \frac{2}{3} N\mu \left(\frac{\mu B}{kT}\right) = \frac{2N\mu^2 B}{3k} \frac{1}{T}$

$\frac{\mu B}{kT} \to \infty$: $\bar{M} \to \frac{2N\mu \cancel{e^{\mu B/kT}}}{1 + 2\cancel{e^{\mu B/kT}}} \to N\mu$

very large



Curie's law

(8)

(a) $U = \sum_n \epsilon \bar{n}_{pl} = \sum_n \epsilon \frac{1}{e^{\epsilon/kT} - 1}$

$$= \sum_n \frac{hcn}{2L} \frac{1}{e^{hcn/2LkT} - 1}$$

$$= \int_0^\infty \frac{hc}{2L} \frac{n\,dn}{e^{hcn/2LkT} - 1} \qquad x \equiv \frac{hcn}{2LkT}$$

$$= \int_0^\infty \frac{hc}{2L}\left(\frac{2LkT}{hc}\right)^2 \frac{x\,dx}{e^x - 1}$$

$$= \frac{2L}{hc}(kT)^2 \underbrace{\int_0^\infty \frac{x\,dx}{e^x - 1}}_{=\pi^2/6} = \frac{\pi^2 L}{3hc}(kT)^2$$

(b) $C_v = \frac{dU}{dT} = \frac{2\pi^2 L k^2}{3hc} T$

(c) $S = \int_0^T \frac{C_v}{T} dT = \frac{2\pi^2 L k^2}{3hc} \underbrace{\int_0^T 1\, dT}_{=T} = \frac{2\pi^2 k^2}{3hc} \underline{\underline{LT}}$

(d) S is kept constant. Therefore

$LT = (1\ \text{m})(3000\ \text{K}) = L \cdot (3\ \text{K})$.

$\therefore L = 1000\ \text{m} = 1\ \text{km}$