PHY 280F **Midterm Exam** **Fall 2005**

Thursday, October 20th

PART I: Provide short answers to the following questions. You will not need more than one or two sentences to answer these questions. You do not need to provide a lengthy explanation.

1. State the second law of thermodynamics in two different systems. First, consider a system completely isolated from the environment. Secondly, consider a system in equilibrium with a thermal reservoir at constant temperature T. (*6 points*)

2. A one-dimensional random walk is a journey consisting of N steps, all the same size, each chosen randomly to be either forward or backward. Where are you most likely to find yourself at the end of a long random walk? (*6 points*)

3. Consider two different monatomic ideal gases, each with the same energy (U), volume (V), and number of particles (N). They occupy the two halves of a divided container, separated by a partition.

   (a) If we now remove the partition, what is total the entropy change? Assume that the container is isolated from the world. (*3 points*)

   (b) What is the entropy change if we start with the *same* gases? (*3 points*)

4. George is doing an infrared absorption spectroscopy experiment with a newly discovered atom called qarium. Calculation shows that this atom's first excited state has an energy of $4.3\text{X}10^{-4}$ eV (above the ground state). In his measurement, George found that the ratio of the atoms in the ground state and those in the first excited state is roughly 2.7 ($\approx e$). However, he also found out that his thermometer in the apparatus was malfunctioning during the experiment. To account for his data, George suggested that the atoms must be in thermal equilibrium at a certain temperature. What is that temperature? (Find the numerical value for the temperature. This problem involves a simple calculation, which does not require a calculator.) (*6 points*)

5. Hydrogen gas can be considered as a diatomic ideal gas with 3 translational, 2 rotational, and 2 vibrational degrees of freedom. Here the 2 vibrational degrees of freedom include both potential and kinetic energy terms.

   (a) According to the equipartition theorem, what is the heat capacity (at constant volume) per hydrogen molecule? (*3 points*)

   (b) However, experimentally it was found that at room temperature the heat capacity of hydrogen is much smaller than this value. Why? (*3 points*)

PART II: The following questions require mathematical derivations. Provide all the mathematical steps leading to your answer.

6. (*Total 30 points*) The nucleus of the nitrogen isotope ^{14}N acts, in some ways, like a spinning sphere of positive charge. The nucleus has a magnetic moment of *m*. Consider such a nucleus to be spatially fixed (therefore, distinguishable), but free to take on various orientations relative to an external inhomogeneous electric field. In other words, the nucleus has three energy states, each with a definite value for the magnetic moment along the field direction. The spin orientations and the associated energies are the following: spin up ($m = +\mu; E = \varepsilon_0$); spin "sideways" ($m = 0; E = 0$); spin down ($m = -\mu; E = \varepsilon_0$). Note that the energy of the spin down state is the same as the spin up state. Here ε_0 denotes a small positive energy.

   (a) Now, the system reaches a thermal equilibrium at temperature *T*. What is the probability of finding the nucleus with spin up? In what limit would this be 1/3? (5 points)

   (b) Calculate the average total energy of *N* nuclei. (5 points)

   (c) Find the Helmholtz free energy of *N* nuclei. (5 points)

   (d) Using the Helmholtz free energy obtained above, find the entropy of *N* nuclei in the limit of high temperature. Is the result what you would expect? Why? (10 points)

   (e) What is the average magnetic moment $\overline{m}$? Give a qualitative reason for your numerical result. (5 points)

7. (Total 40 points) One of the examples extensively considered in the class is the Einstein model of solid. This model does not describe real solids very well at low temperatures. In this problem, we will consider a one-dimensional solid and compare the Einstein model and Debye model.

   First, let's consider the Einstein model, which is a collection of independent oscillators. These oscillators can take energies in multiples of $\varepsilon = hf$ only, where *h* is the Planck's constant. In the one-dimensional Einstein solid, there are *N* such oscillators, and the total energy is *U*. Note that the total number of energy units, *q*, is just U / ε. Both *N* and *q* are large numbers and in the limit of low temperature, $N >> q$.

   (a) What is the multiplicity of this Einstein solid in the low-temperature limit. Use Stirling's approximation to simplify your result. What is the entropy of the Einstein solid in the low-temperature limit? (7 points)

(b) Starting from the above entropy, find a formula for the temperature of an Einstein solid in this temperature limit. Solve this formula to find the energy *U* as a function of temperature. (7 points)

(c) Calculate the temperature dependence of the heat capacity of an Einstein solid in the low-temperature limit. (5 points)

Next, we will consider the Debye model of a solid. The problem with the Einstein model is that the atoms in a real crystal do not vibrate independently of each other. If you wiggle one atom, its neighbors will also start to wiggle. In the one-dimensional Debye model, there are N atoms in the solid whose length is *L*. In this model, the oscillation of atoms can be described by different modes of oscillation, just like the modes of electromagnetic waves in a cavity. Each mode of oscillation has a set of equally spaced energy levels $\varepsilon = hf = \frac{hc_s n}{2L}$, where n=0,1,2,…, N and c_s is the sound velocity.

(d) Using Planck's distribution, write the formula for the total energy in this Debye model of solid. Consider only one polarization. Do not carry out the summation. (5 points)

(e) Now, convert the sum in part (d) to an integral, and simplify this integral through the following change of variable: $x \equiv \frac{hc_s}{2LkT} n$. Further simplify your answer by using the one dimensional Debye temperature $\Theta_D \equiv \frac{hc_s N}{2Lk}$. In the limit of low temperature, the integral can be carried out. (Consult the formulae sheet provided). Find the total energy *U* of the Debye solid in the low-temperature limit. (6 points)

(f) Find the temperature dependence of constant-volume heat capacity C_V using the result obtained in part (e). (5 points)

(g) Plot qualitatively the results of part (c) and (f). Plot the heat capacity C_V as a function of temperature *T*. (5 points)