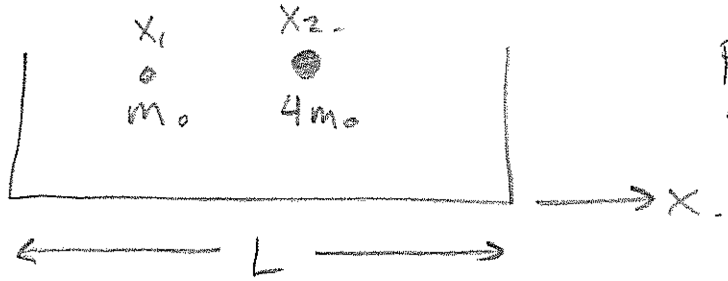
Suggested Ch 13 problems 13.1, 13.2, 13.6(a).

$\boxed{13.1}$



particle A = m_o, position = x_1
particle B = $4m_o$, position = x_2

Schrödinger Eqn: (eq. 13-2)

$$-\frac{\hbar^2}{2m_o}\frac{\partial^2\Psi}{\partial x_1^2} - \frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x_2^2} + V_1(x_1)\Psi + V_2(x_2)\Psi = i\hbar\frac{\partial\Psi}{\partial t}$$

assume a separable solution, as in text:

$$\Psi(x_1, x_2, t) = \psi_A(x_1)\cdot\psi_B(x_2)\cdot f(t).$$

$$\Rightarrow \left[-\frac{\hbar^2}{2m_1}\frac{\partial^2\psi_A}{\partial x_1^2} + V_1(x_1)\psi_A\right]\psi_B f$$

$$+ \left[-\frac{\hbar^2}{2m}\frac{\partial^2\psi_B}{\partial x_2^2} + V_2(x_2)\psi_B\right]\psi_A f = i\hbar\psi_A\psi_B\frac{\partial f}{\partial t}$$

We can set:

$$\left.\begin{aligned} &(1)\quad -\frac{\hbar^2}{2m_1}\frac{\partial^2\psi_A}{\partial x_1^2} + V_1(x_1)\psi_A = E_A\psi_A. \\ &(2)\quad -\frac{\hbar^2}{2m_1}\frac{\partial^2\psi_A}{\partial x_2^2} + V_2(x_1)\psi_B = E_B\psi_B \end{aligned}\right\}\ \text{total energy} = E_A + E_B.$$

$$(E_A + E_B)f = i\hbar\frac{\partial f}{\partial t} \Rightarrow f(t) = e^{-i(E_A+E_B)t/\hbar}.$$

↳ gives our time-dependence.

From (1) and (2), find E_A, E_B!

For a 1-D box, we know $E = \dfrac{\hbar^2 n^2}{2mL^2}$

$$\Rightarrow E_A = \frac{\hbar^2 n_A^2}{2m_oL^2}$$

$$E_B = \frac{\hbar^2 n_B^2}{2(4m_o)L^2} = \frac{\hbar^2 n_B^2}{8m_oL^2}.$$

$$E_{tot} = E_A + E_B = \frac{\hbar^2}{2m_oL^2}\left(n_A^2 + \left(\frac{n_B}{2}\right)^2\right) \; ; \; n_A, n_B = 1, 2, 3 \ldots$$

What about energy degeneracy?

n_A	n_B	$E/\left(\frac{\hbar^2}{2m_oL^2}\right)$	degeneracy
1	1	1.25	1
1	2	2	1
1	3	3.25	1
1 2	4 2	5	2
1	5	7.25	1
1 3	6 2	10	2

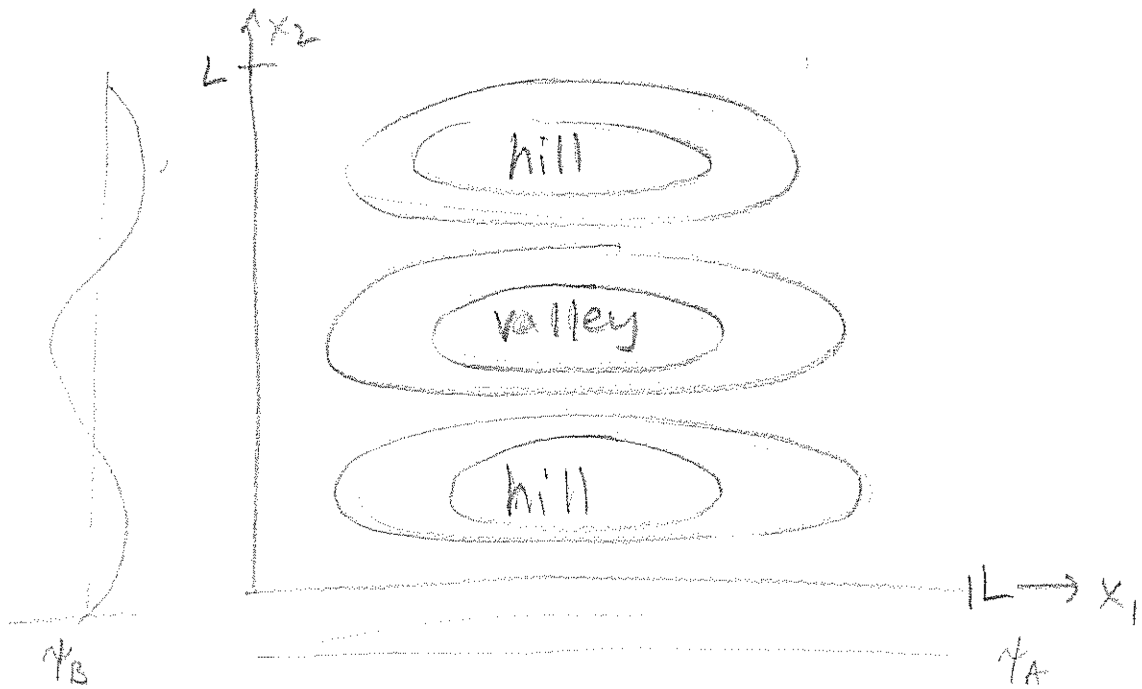
etc...

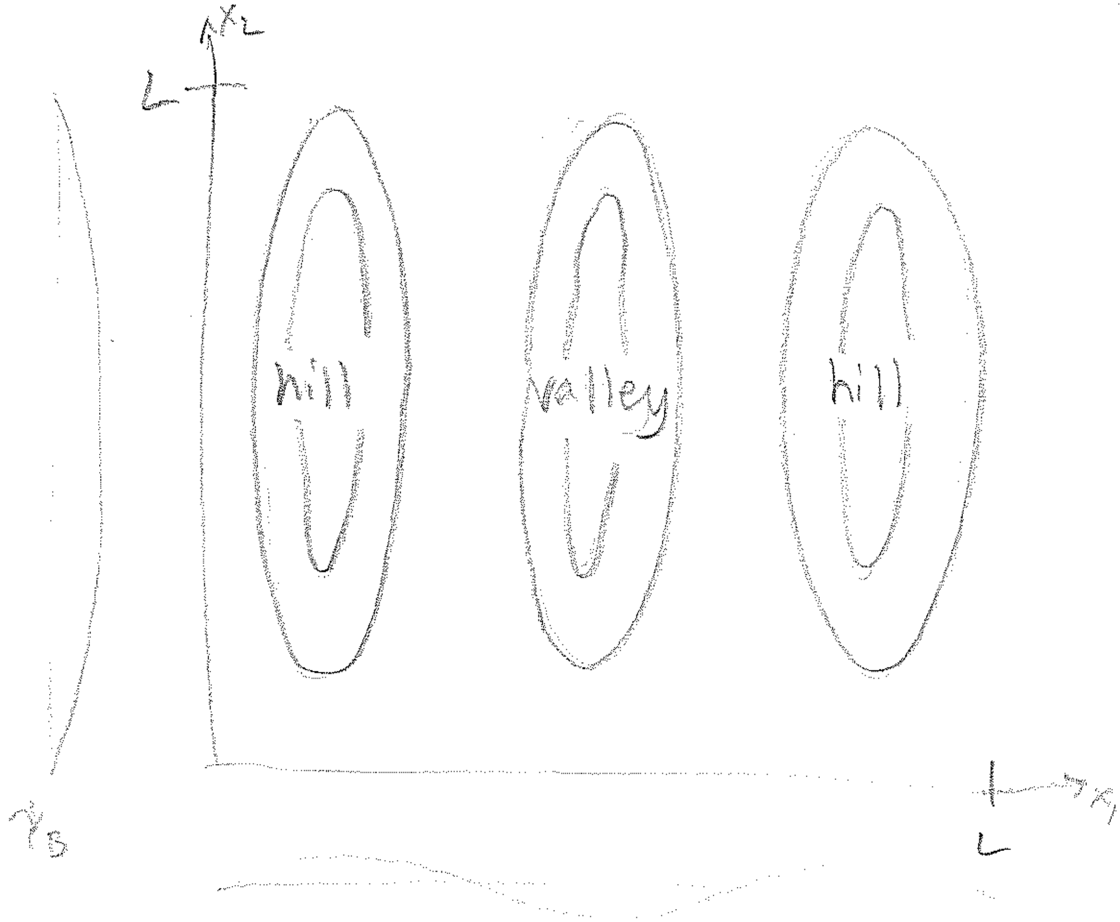
13.2

a) $\psi(x_1, x_2) = \frac{2}{L} \sin\left(\frac{\pi x_1}{L}\right) \sin\left(\frac{3\pi x_2}{L}\right)$

or

$\psi(x_1, x_2) = \frac{2}{L} \sin\left(\frac{3\pi x_1}{L}\right) \sin\left(\frac{\pi x_2}{L}\right)$

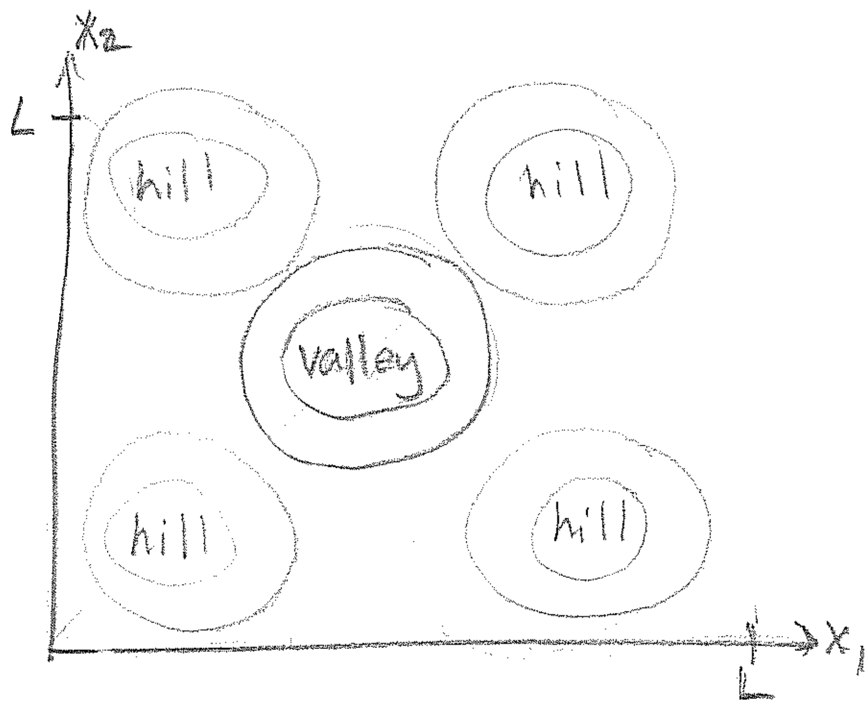




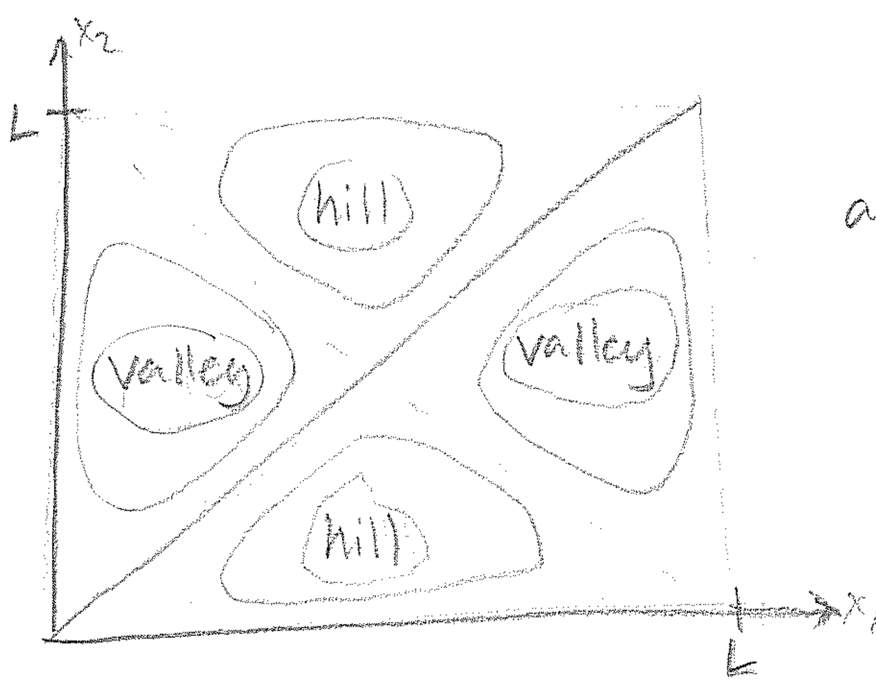
b) identical particles:

symmetric: $\psi_S(x_1, x_2) = \frac{1}{\sqrt{2}}\left(\frac{2}{L}\right)\left[\sin\left(\frac{\pi x_1}{L}\right)\sin\left(\frac{3\pi x_2}{L}\right) + \sin\left(\frac{\pi x_2}{L}\right)\sin\left(\frac{3\pi x_1}{L}\right)\right]$

antisymmetric: $\psi_A(x_1, x_2) = \frac{1}{\sqrt{2}}\left(\frac{2}{L}\right)\left[\sin\left(\frac{\pi x_1}{L}\right)\sin\left(\frac{3\pi x_2}{L}\right) - \sin\left(\frac{\pi x_2}{L}\right)\sin\left(\frac{3\pi x_1}{L}\right)\right]$



symmetric (add 2 graphs from (a)).



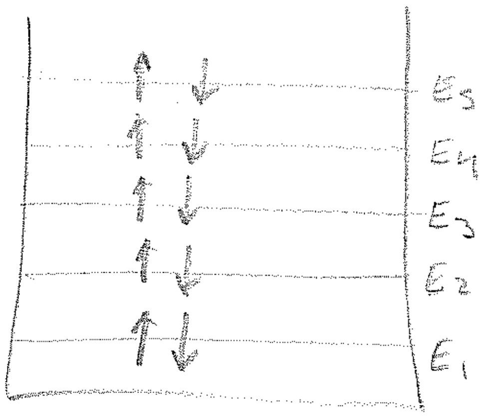
antisymmetric

13,6a)

10 electrons, 1 box.

→ no 2 electrons may have the same state.

eg) cannot put 2 ↑ electrons in same energy level/eigenstate.

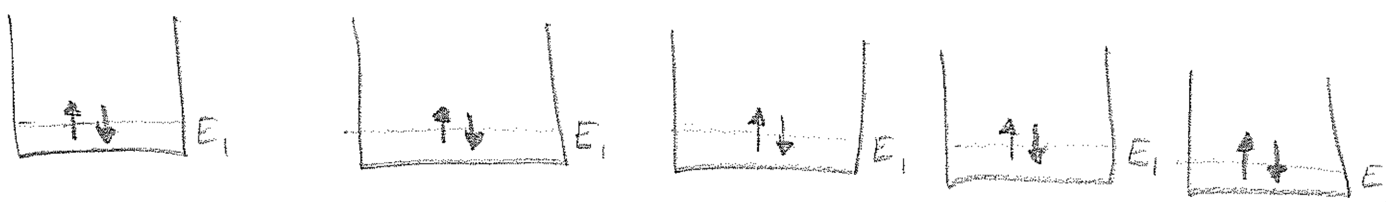


$$E_{tot} = 2(E_1 + E_2 + E_3 + E_4 + E_5)$$

$$= \frac{2\hbar^2}{2mL^2}(1^2 + 2^2 + 3^2 + 4^2 + 5^2)$$

$$= \frac{\hbar^2}{mL^2}(55).$$

→ 10 electrons, 5 boxes;



$$E_{tot} = 5(2(E_1)) = \frac{10\hbar^2(1)^2}{2mL^2} = \frac{5\hbar^2}{mL^2}$$

⇒ total energy is $\frac{55}{5} = 11$ times higher if all in one box.