PHY291 Assignment 3

Quantum
PS # 7

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9-6.

a) $\psi(x,t) = \frac{1}{\sqrt{2}} \cdot \sqrt{\frac{2}{L}} \sin\left(\frac{\pi x}{L}\right) e^{-i\omega_1 t} + \frac{1}{\sqrt{2}} \cdot \sqrt{\frac{2}{L}} \sin\left(\frac{2\pi x}{L}\right) e^{-i\omega_2 t}$

$\psi^*(x,t) = \frac{1}{\sqrt{2}} \cdot \sqrt{\frac{2}{L}} \sin\left(\frac{\pi x}{L}\right) e^{i\omega_1 t} + \frac{1}{\sqrt{2}} \cdot \sqrt{\frac{2}{L}} \sin\left(\frac{2\pi x}{L}\right) e^{i\omega_2 t}$

$\frac{d\psi}{dx} = \frac{1}{\sqrt{2}} \cdot \sqrt{\frac{2}{L}} \cdot \frac{\pi}{L} \cos\left(\frac{\pi x}{L}\right) e^{-i\omega_1 t} + \frac{1}{\sqrt{2}} \cdot \sqrt{\frac{2}{L}} \cdot \frac{2\pi}{L} \cos\left(\frac{2\pi x}{L}\right) e^{-i\omega_2 t}$

$\frac{d\psi^*}{dx} = \frac{1}{\sqrt{2}} \cdot \sqrt{\frac{2}{L}} \cdot \frac{\pi}{L} \cos\left(\frac{\pi x}{L}\right) e^{i\omega_1 t} + \frac{1}{\sqrt{2}} \cdot \sqrt{\frac{2}{L}} \cdot \frac{2\pi}{L} \cos\left(\frac{2\pi x}{L}\right) e^{i\omega_2 t}$

$J(x,t) = \frac{-i\hbar}{2m}\left(\psi^* \frac{d\psi}{dx} - \psi \frac{d\psi^*}{dx}\right)$

$= \frac{-i\hbar}{2m}\Big[\left(\frac{1}{\sqrt{L}} \sin\left(\frac{\pi x}{L}\right) e^{i\omega_1 t} + \frac{1}{\sqrt{L}} \sin\left(\frac{2\pi x}{L}\right) e^{i\omega_2 t}\right)\Big(\frac{1}{\sqrt{L}} \cdot \frac{\pi}{L} \cos\left(\frac{\pi x}{L}\right) e^{-i\omega_1 t}$

$+ \frac{1}{\sqrt{L}} \cdot \frac{2\pi}{L} \cos\left(\frac{2\pi x}{L}\right) e^{-i\omega_2 t}\Big) - \Big(\frac{1}{\sqrt{L}} \sin\left(\frac{\pi x}{L}\right) e^{-i\omega_1 t}$

$+ \frac{1}{\sqrt{L}} \sin\left(\frac{2\pi x}{L}\right) e^{-i\omega_2 t}\Big)\Big(\frac{1}{\sqrt{L}} \cdot \frac{\pi}{L} \cos\left(\frac{\pi x}{L}\right) e^{i\omega_1 t}$

$+ \frac{1}{\sqrt{L}} \cdot \frac{2\pi}{L} \cos\left(\frac{2\pi x}{L}\right) e^{i\omega_2 t}\Big)\Big]$

$= \frac{-i\hbar}{2m}\Big[\frac{\pi}{L^2} \sin\left(\frac{\pi x}{L}\right)\cos\left(\frac{\pi x}{L}\right) + \frac{2\pi}{L^2} \sin\left(\frac{\pi x}{L}\right)\cos\left(\frac{2\pi x}{L}\right) e^{-i(\omega_2-\omega_1)t}$

$+ \frac{\pi}{L^2} \sin\left(\frac{2\pi x}{L}\right)\cos\left(\frac{\pi x}{L}\right) e^{i(\omega_2-\omega_1)t} + \frac{2\pi}{L^2} \sin\left(\frac{2\pi x}{L}\right)\cos\left(\frac{2\pi x}{L}\right)$

$- \frac{\pi}{L^2} \sin\left(\frac{\pi x}{L}\right)\cos\left(\frac{\pi x}{L}\right) - \frac{2\pi}{L^2} \sin\left(\frac{\pi x}{L}\right)\cos\left(\frac{2\pi x}{L}\right) e^{i(\omega_2-\omega_1)t}$

$- \frac{\pi}{L^2} \sin\left(\frac{2\pi x}{L}\right)\cos\left(\frac{\pi x}{L}\right) e^{-i(\omega_2-\omega_1)t} - \frac{2\pi}{L^2} \sin\left(\frac{2\pi x}{L}\right)\cos\left(\frac{2\pi x}{L}\right)$

$= \frac{-i\hbar\pi}{2mL^2}\left[\sin\left(\frac{2\pi x}{L}\right)\cos\left(\frac{\pi x}{L}\right) - 2\sin\left(\frac{\pi x}{L}\right)\cos\left(\frac{2\pi x}{L}\right)\right] \cdot \left(e^{i(\omega_2-\omega_1)t} - e^{-i(\omega_2-\omega_1)t}\right)$

$$\boxed{J(x,t) = \frac{\hbar}{mL^2} \sin^3\left(\frac{\pi x}{L}\right) \sin\left((\omega_2-\omega_1)t\right)}$$

at $x = L/2$

$$\boxed{J(x,t) = \frac{h}{mL^2} \sin((\omega_2-\omega_1)t)}$$

b) $P = \frac{1}{2} + \left(\frac{4}{3\pi}\right)\cos\left[\frac{(E_2-E_1)t}{\hbar}\right]$

$$\frac{dP}{dt} = \frac{-4}{3\pi}\frac{(E_2-E_1)}{\hbar}\sin\left(\frac{(E_2-E_1)t}{\hbar}\right)$$

$$\frac{dP}{dt} = -\frac{4}{3\pi}\frac{(4E_1-E_1)}{\hbar}\sin((\omega_2-\omega_1)t)$$

$$= -\frac{4}{\cancel{3}\pi}\cdot\frac{\cancel{3}E_1}{\hbar}\sin((\omega_2-\omega_1)t)$$

$$= -\frac{4}{\pi}\frac{h^2}{8mL^2}\cdot\frac{1}{\hbar}\sin((\omega_2-\omega_1)t)$$

$$= \frac{-h}{mL^2}\sin((\omega_2-\omega_1)t)$$

$$= -J(x=\tfrac{L}{2},t)$$

Quantum
Assignment #2

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1-8

$$J(x,t) = -\frac{i\hbar}{2m}\left[\Psi^* \frac{d\Psi}{dx} - \Psi \frac{d\Psi^*}{dx}\right]$$

a) $\Psi(x_0,t_0) = 0 \Rightarrow \Psi^*(x_0,t_0) = 0$

$\therefore J(x,t) = -\frac{i\hbar}{2m}(0-0) = 0$ so its sufficient.

b) $|\Psi(x_0,t_0)|^2 = |\Psi(x_0,t)|^2$, for all t.

let $\Psi(x,t) = Ae^{-ikx-i\omega t}$

$\frac{d\Psi}{dx} = ikA\Psi(x,t)$

$\Psi^*(x,t) = Ae^{i\omega t + ikx}$

$\frac{d\Psi^*}{dx} = -ikA\Psi^*(x,t)$

$\therefore J = -\frac{i\hbar}{2m}\left[(Ae^{i\omega t-ikx})(ikAe^{ikx-i\omega t}) - (Ae^{ikx-i\omega t})(-ikAe^{i\omega t-ikx})\right]$

$= -\frac{i\hbar}{2m}\left[ik|A|^2 + ik|A|^2\right]$

$= \frac{\hbar k}{h}|A|^2 = J(x_0,t_0) \neq 0$ so not sufficient.

c) based on above, condition $|\Psi(x_0,t_0)|^2 = |\Psi(x,t_0)|^2$ would be insufficient to ensure $J=0$.

d) $\Psi(x_0+b,t) = \Psi(x_0-b,t)$, for all $b>0$

$\Rightarrow \frac{d\Psi}{dx} = 0$ and $\frac{d\Psi^*}{dx} = 0$

since Ψ is symmetrical about the x axis

$\therefore J = -\frac{i\hbar}{2m}\left[\Psi^*(0) - \Psi(0)\right] \Rightarrow J = 0$

so the condition is sufficient

e) $\Psi(x=x_0, t=t_0)$ is real

$\Psi = A\sin(k_0x - \omega t_0) = 0$ only when $kx_0 - \omega t_0 = 2n\pi$

so not all values of (x_0,t_0) would produce $J=0$.

so the condition is insufficient.

Quantum

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9-14

$$T \approx 16\left(\frac{E}{V_0}\right)\left(1-\frac{E}{V_0}\right)e^{-2\alpha L}$$

$E = 2eV \quad V_0 = 4eV \quad L = 10A°$

$$\alpha = \frac{[2m(V_0 - E)]^{1/2}}{\hbar}$$

$$\alpha = \frac{[2 * 9.109\times10^{-31} \cdot (4-2) \cdot 1.6\times10^{-19}]^{1/2}}{1.054\times10^{-34}}$$

$\alpha = 7.2\times10^{9}\ m^{-1}$

$\alpha L = 7.2\times10^{9}\ m^{-1} \times 10\times10^{-10}\ m$

$\alpha L = 7.24$

so $T = 16\left(\frac{2}{4}\right)\left(1-\frac{2}{4}\right)e^{-2(7.24)}$

so $\boxed{T = 2.06\times10^{-6}}$

9-18

a) $J = \frac{-i\hbar}{2m}\left[\psi^* \frac{d\psi}{dx} - \psi \frac{d\psi^*}{dx}\right]$

$\psi(x) = Be^{-\alpha x} + Ce^{\alpha x}$

$\psi^*(x) = B^* e^{-\alpha x} + C^* e^{\alpha x}$

$\frac{d\psi}{dx} = -\alpha Be^{-\alpha x} + \alpha Ce^{\alpha x}$

$\frac{d\psi^*}{dx} = -\alpha B^* e^{-\alpha x} + \alpha C^* e^{\alpha x}$

So $J = \frac{-i\hbar}{2m}\Big[(B^* e^{-\alpha x} + C^* e^{\alpha x})(-\alpha Be^{-\alpha x} + \alpha Ce^{\alpha x})$

$- (Be^{-\alpha x} + Ce^{\alpha x})(-\alpha B^* e^{-\alpha x} + \alpha C^* e^{\alpha x})\Big]$

$J = \frac{-i\hbar}{2m}\Big[-\alpha B^* B e^{-2\alpha x} + \alpha B^* C - \alpha BC^* + \alpha C^* C e^{2\alpha x}$

$+ \alpha BB^* e^{-2\alpha x} - \alpha BC^* + \alpha B^* C - \alpha CC^* e^{2\alpha x}\Big]$

$J = -\frac{i\hbar\alpha}{2m}\left[2B^*C - 2BC^*\right]$

$\boxed{J = \frac{-i\hbar\alpha}{m}\left[B^*C - BC^*\right]}$

b) at $x = L$

① $De^{ikL} = Be^{-\alpha L} + Ce^{\alpha L}$

② $ikDe^{ikL} = -\alpha Be^{-\alpha L} + \alpha Ce^{\alpha L}$

① − ② $\cdot 1/\alpha$

$De^{ikL} - \frac{ik}{\alpha} De^{ikL} = 2Be^{-\alpha L}$

$\Rightarrow B = \frac{D}{2}\left(1 - \frac{ik}{\alpha}\right) e^{(ik+\alpha)L}$

$① + ② \cdot \frac{1}{\alpha}$

$$De^{ikL} + \frac{ik}{\alpha} De^{ikL} = 2Ce^{\alpha L}$$

$$\text{so } C = \frac{D}{2}\left(1 + \frac{ik}{\alpha}\right) e^{(ik-\alpha)L}$$

$$J = -\frac{i\hbar\alpha}{m}\left[\left(\frac{D}{2}\left(1+\frac{ik}{\alpha}\right) e^{(ik-\alpha)L}\right)\left(\frac{D}{2}\left(1+\frac{ik}{2}\right) e^{(ik-\alpha)L}\right) - \left(\frac{D}{2}\left(1-\frac{ik}{\alpha}\right) e^{(ik+\alpha)L}\right)\left(\frac{D}{2}\left(1-\frac{ik}{\alpha}\right) e^{(-ik-\alpha)L}\right)\right]$$

$$J = -\frac{i\hbar\alpha}{m}\left[\frac{D^2}{4}\left(1+\frac{ik}{\alpha}\right)^2 - \frac{D^2}{4}\left(1-\frac{ik}{\alpha}\right)^2\right]$$

$$J = \frac{-i\hbar\alpha|D|^2}{4m}\left[1 + \frac{2ik}{\alpha} + \frac{i^2k^2}{\alpha^2} - 1 + \frac{2ik}{\alpha} - \frac{i^2k^2}{\alpha^2}\right]$$

$$J = \frac{-i\hbar\alpha|D|^2}{4m}\left[\frac{4ik}{\alpha}\right]$$

$$\boxed{J = \frac{\hbar k}{m} \cdot |D|^2}$$

Quantum

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10-1

phonograph $R = 15cm = 0.15m$

$m = 0.05kg$

$$\text{Inertia} = \frac{1}{2}MR^2 = \frac{1}{2}(0.05kg)(0.15m)^2$$

$$= 5.625 \times 10^{-4} kg \cdot m^2$$

$L = I\omega \quad L = 10^4 \hbar$

$$\omega = \frac{L}{I} = \frac{10^4 \cdot \hbar}{5.625 \times 10^{-4}} = 1.873 \times 10^{-22}$$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{1.873 \times 10^{-22}} = 3.35 \times 10^{22} \text{ sec}$$

$$\boxed{T \simeq 1 \times 10^{15} \text{ years}}$$

Molecules 2-A

1) $\lambda = 4cm = 0.04m$ $\quad m_{NA} = 126 \times 2 \times 10^{-27} kg$

$= 4.6 \times 10^{-26} kg$

$\ell = 1$

$$E = \frac{\ell \hbar^2}{M r_0^2} = \frac{hc}{\lambda}$$

$$= \frac{\ell h^2}{4\pi^2 M} \cdot \frac{\lambda}{hc} = r_0^2 \qquad r_0 = \frac{1}{2\pi}\sqrt{\frac{\ell h \lambda}{Mc}}$$

$$r_0 = \frac{1}{2\pi}\sqrt{\frac{1 \times 6.626 \times 10^{-34} \times 0.04m}{4.6 \times 10^{-26} \times 3 \times 10^8}} \qquad \boxed{r_0 = 2.2 \text{Å}}$$

ii) $r_0 = 1Å = 10^{-10}$ m

$M = 2 \times 2 \times 10^{-27}$
$= 4 \times 10^{-27}$ kg $l = 1$

$\Delta E_L = \frac{l \hbar^2}{M r_0^2} = \frac{1 \times (1.054 \times 10^{-34})^2}{4 \times 10^{-27} kg \cdot (10^{-10})^2}$
$= 2.78 \times 10^{-22}$

$\boxed{\Delta E_l = 1.73 \times 10^{-3} eV}$

$\Delta E_l = \frac{hc}{\lambda}$

$\lambda = \frac{hc}{\Delta E_l} = \frac{6.626 \times 10^{-34} \times 3 \times 10^8 m/s}{2.78 \times 10^{-22} J}$

$\boxed{\lambda = 0.7 mm}$ vs the size of the hole.

iii) $r_0 = 2.2Å = 2.2 \times 10^{-10}$ m $T = 300C$
$m_{NA} = 4.6 \times 10^{-26}$ kg $= 573K$

$\Delta E_l = kT$
$\Delta E_l = 1.381 \times 10^{-23} \times 573$
$= 7.91 \times 10^{-21}$ J
$\Delta E_l = \frac{l(l+1)\hbar^2}{M r_0^2}$ $l^2 + l - \frac{\Delta E_l M r_0^2}{\hbar^2} = 0$

$l^2 \approx \frac{\Delta E_l M r_0^2}{\hbar^2}$

so $l = \sqrt{\frac{7.91 \times 10^{-21} \times 4.6 \times 10^{-26} \times (2.2 \times 10^{-10})^2}{(1.054 \times 10^{-34})^2}}$

$\boxed{l \approx 40}$