Simple Gravity Model Example

Given:

O/D	1	2	Total
1			200
2			300
Total	100	400	500

Travel Times (tij):

i \ j	1	2
1	5	15
2	12	6

Impedance Function $f_{ij} = 1 / t_{ij}^2$

i \ j	1	2
1	0.04000	0.00444
2	0.00694	0.02778

f_{ij}

Iteration k=1: ($D^{*1}_j = D_j$)

D^{*1}_j= 100 400

Denominator, Origin i=1 5.78 ← $\Sigma_{j'} D^{*k}_{j'} f_{1j'}$ (0.0400)(100) + (0.00444)(400) = 5.776.

Denominator, Origin i=2 11.81 ← $\Sigma_{j'} D^{*k}_{j'} f_{2j'}$ (0.00694)(100) + (0.02778)(400) = 11.806

i \ j	1	2	Total
1	138.5	61.5	200.0
2	17.6	282.4	300.0
Total	156.1	343.9	500

← $T_{ij}^k = O_i D^{*k}_j f_{ij} / \Sigma_{j'} D^{*k}_{j'} f_{ij'}$

Rj= 0.641 1.163 ← $D_j / \Sigma_i T_{ij}^k$ $\frac{100}{156.1}$ $\frac{400}{343.9}$

|Rj-1|= 0.359 0.163 $\left|\frac{100}{156.1}-1\right|$ $\left|\frac{400}{343.9}-1\right|$

Iteration k=2: (D*j(new)= D*j(old)*Rj) 100 × 0.641 400 × 1.163

D^{*2}_j= 64.1 465.3

Denominator, Origin i=1 4.63

Denominator, Origin i=2 13.37

i \ j	1	2	Total
1	110.7	89.3	200.0
2	10.0	290.0	300.0
Total	120.7	379.3	500

Rj= 0.829 1.054

|Rj-1|= 0.171 0.054

Iteration k=3: (D*j(new)= D*j(old)*Rj)

D^{*3}_j= 53.1 490.6

Denominator, Origin i=1 4.30

Denominator, Origin i=2 14.00

i \ j	1	2	Total
1	98.7	101.3	200.0
2	7.9	292.1	300.0
Total	106.6	393.4	500

Rj= 0.938 1.017

|Rj-1|= 0.062 0.017

Iteration k=4: (D*j(new)= D*j(old)*Rj)

D^{*4}_j= 49.8 498.8

Denominator, Origin i=1 4.21

Denominator, Origin i=2 14.20

i \ j	1	2	Total
1	94.7	105.3	200.0
2	7.3	292.7	300.0
Total	102.0	398.0	500

Rj= 0.981 1.005

|Rj-1|= 0.019 0.005

Assume epsilon of 2% - converged

Simple Gravity Model Example

Given:

O/D:	1	2	Total
1			200
2			300
Total	100	400	500

Travel Times (tij):

O/D:	1	2
1	5	15
2	12	6

Impedance Function = (tij)^(-2)

O/D:	1	2
1	0.04000	0.00444
2	0.00694	0.02778

USING BALANCING PROCEDURE FROM FLOWCHART

Iteration 1: (D*j = Dj)

D*j= 100 400

Denominator, Org=1 5.78

Denominator, Org=2 11.81

O/D:	1	2	Total
1	138.5	61.5	200.0
2	17.6	282.4	300.0
Total	156.1	343.9	500

Rj= 0.641 1.163

Iteration 2: (D*j(new)= D*j(old)*Rj)

D*j= 64.05797 465.2632

Denominator, Org=1 4.63

Denominator, Org=2 13.37

O/D:	1	2	Total
1	110.7	89.3	200.0
2	10.0	290.0	300.0
Total	120.7	379.3	500

Rj= 0.829 1.054

Iteration 3: (D*j(new)= D*j(old)*Rj)

D*j= 53.08873 490.6055

Denominator, Org=1 4.30

Denominator, Org=2 14.00

O/D:	1	2	Total
1	98.7	101.3	200.0
2	7.9	292.1	300.0
Total	106.6	393.4	500

Rj= 0.938 1.017

Iteration 4: (D*j(new)= D*j(old)*Rj)

D*j= 49.81137 498.8104

Denominator, Org=1 4.21

Denominator, Org=2 14.20

O/D:	1	2	Total
1	94.7	105.3	200.0
2	7.3	292.7	300.0
Total	102.0	398.0	500

Rj= 0.981 1.005

Assume epsilon of 2% - converged

USING ENTROPY FORMULATION

Iteration1 (Bj = 1)

Bj = 1 1

Denom A1 5.78

Denom A2 11.81

O/D:	1	2	Total	Ai
1	138.5	61.5	200.0	0.173077
2	17.6	282.4	300.0	0.084706
Total	156.1	343.9	500	

Iteration2

Bj = 0.64058 1.163158

Denom A1 4.63

Denom A2 13.37

O/D:	1	2	Total	Ai
1	110.7	89.3	200.0	0.215975
2	10.0	290.0	300.0	0.074801
Total	120.7	379.3	500	

Iteration3

Bj = 0.530887 1.226514

Denom A1 4.30

Denom A2 14.00

O/D:	1	2	Total	Ai
1	98.7	101.3	200.0	0.232341
2	7.9	292.1	300.0	0.071446
Total	106.6	393.4	500	

Iteration 4

Bj = 0.498114 1.247026

Denom A1 4.21

Denom A2 14.20

O/D:	1	2	Total	Ai
1	94.7	105.3	200.0	0.237564
2	7.3	292.7	300.0	0.070414
Total	102.0	398.0	500	

Biproportional Updating, Alternate Row/Column Balancing

Future Year Trip Origins Destinations

O/D:	1	2	Total
1			200
2			300
Total	100	400	500

Base Year Matrix

O/D:	1	2	Total
1	60	90	150
2	30	220	250
Total	90	310	400

Balance Rows:

O/D:	1	2	Total
1	80	120	200
2	36	264	300
Total	116	384	500
Rj	0.86	1.04	

Balance Columns:

O/D:	1	2	Total	Ri
1	69.0	125.0	194.0	1.03
2	31.0	275.0	306.0	0.98
Total	100	400	500	

Balance Rows:

O/D:	1	2	Total
1	71.1	128.9	200
2	30.4	269.6	300
Total	101.5	398.5	500
Rj	0.98	1.00	

Balance Columns:

O/D:	1	2	Total	Ri
1	70.0	129.4	199.4	1.00
2	30.0	270.6	300.6	1.00
Total	100	400	500	

Balance Rows:

O/D:	1	2	Total
1	70.2	129.8	200
2	29.9	270.1	300
Total	100.1	399.9	500
Rj	1.00	1.00	

Balance Columns:

O/D:	1	2	Total	Ri
1	70.1	129.8	199.9	1.00
2	29.9	270.2	300.1	1.00
Total	100	400	500	

Converged!

Biproportional Updating, Uisng "Gravity" Formulation.

Given:

O/D:	1	2	Total
1			200
2			300
Total	100	400	500

Base Year Matrix

O/D:	1	2
1	60	90
2	30	220

$T_{ij} = A_i O_i B_j D_j T_{ij}^{0}$? $A_i = \frac{1}{\Sigma_j B_j D_j f_{ij}}$?

Iteration 1: (D*j = Dj)

D*j= 100 400

Denominator, Org=1 42000.00 $= 100(60)+400(90)$

Denominator, Org=2 91000.00 $= 100(30)+400(220)$

$T_{ij}^{*} = \frac{O_i D_j^{*} T_{ij}^{0}}{\Sigma_j D_j^{*} T_{ij}^{0}}$

$\frac{200 \times 100 \times 60}{42000} = 28.6$ $\frac{200 \times 400 \times 90}{42000} = 171.4$

O/D:	1	2	Total
1	28.6	171.4	200.0
2	9.9	290.1	300.0
Total	38.5	461.5	500

Rj= 2.600 0.867

$\frac{D_j}{\Sigma_i T_{ij}^{*}}$ $\frac{100}{38.5}$ $\frac{400}{461.5}$

Iteration 2: (D*j(new)= D*j(old)*Rj)

D*j= 260 346.6667

$100 \times 2.6 = 260$

Denominator, Org=1 46800.00

Denominator, Org=2 84066.67

O/D:	1	2	Total
1	66.7	133.3	200.0
2	27.8	272.2	300.0
Total	94.5	405.5	500

Rj= 1.058 0.986

Iteration 3: (D*j(new)= D*j(old)*Rj)

D*j= 275.1273 341.9661

Denominator, Org=1 47284.59

Denominator, Org=2 83486.36

O/D:	1	2	Total
1	69.8	130.2	200.0
2	29.7	270.3	300.0
Total	99.5	400.5	500

Rj= 1.005 0.999

Iteration 4: (D*j(new)= D*j(old)*Rj)

D*j= 276.5605 341.5236

Denominator, Org=1 47330.76

Denominator, Org=2 83432.01

O/D:	1	2	Total
1	70.1	129.9	200.0
2	29.8	270.2	300.0
Total	100.0	400.0	500

Rj= 1.000 1.000

Converged!