Chapter 7 / Mechanical Properties

A modern Rockwell hardness tester. (Photograph courtesy of Wilson Instruments Division, Instron Corporation, originator of the Rockwell® Hardness Tester.)

Why Study *Mechanical Properties?*

It is incumbent on engineers to understand how the various mechanical properties are measured and what these properties represent; they may be called upon to design structures/components using predetermined materials such that unacceptable levels of deformation and/or failure will not occur. We demonstrate this procedure with respect to the design of a tensile-testing apparatus in Design Example 7.1.

Learning Objectives

After studying this chapter you should be able to do the following:

1. **Define engineering stress and engineering strain.**
2. **State Hooke's law, and note the conditions under which it is valid.**
3. **Define Poisson's ratio.**
4. **Given an engineering stress–strain diagram, determine (a) the modulus of elasticity, (b) the yield strength (0.002 strain offset), and (c) the tensile strength, and (d) estimate the percent elongation.**
5. **For the tensile deformation of a ductile cylindrical specimen, describe changes in specimen profile to the point of fracture.**
6. **Compute ductility in terms of both percent elongation and percent reduction of area for a material that is loaded in tension to fracture.**
7. **Compute the flexural strengths of ceramic rod specimens that have bent to fracture in three-point loading.**
8. **Make schematic plots of the three characteristic stress–strain behaviors observed for polymeric materials.**
9. **Name the two most common hardness-testing techniques; note two differences between them.**
10. **(a) Name and briefly describe the two different microhardness testing techniques, and (b) cite situations for which these techniques are generally used.**
11. **Compute the working stress for a ductile material.**

7.1 Introduction

Many materials, when in service, are subjected to forces or loads; examples include the aluminum alloy from which an airplane wing is constructed and the steel in an automobile axle. In such situations it is necessary to know the characteristics of the material and to design the member from which it is made such that any resulting deformation will not be excessive and fracture will not occur. The mechanical behavior of a material reflects the relationship between its response or deformation to an applied load or force. Important mechanical properties are strength, hardness, ductility, and stiffness.

The mechanical properties of materials are ascertained by performing carefully designed laboratory experiments that replicate as nearly as possible the service conditions. Factors to be considered include the nature of the applied load and its duration, as well as the environmental conditions. It is possible for the load to be tensile, compressive, or shear, and its magnitude may be constant with time, or it may fluctuate continuously. Application time may be only a fraction of a second, or it may extend over a period of many years. Service temperature may be an important factor.

Mechanical properties are of concern to a variety of parties (e.g., producers and consumers of materials, research organizations, government agencies) that have differing interests. Consequently, it is imperative that there be some consistency in the manner in which tests are conducted, and in the interpretation of their results. This consistency is accomplished by using standardized testing techniques. Establishment and publication of these standards are often coordinated by professional societies. In the United States the most active organization is the American Society for Testing and Materials (ASTM). Its *Annual Book of ASTM Standards* comprises numerous volumes, which are issued and updated yearly; a large number of these standards relate to mechanical testing techniques. Several of these are referenced by footnote in this and subsequent chapters.

The role of structural engineers is to determine stresses and stress distributions within members that are subjected to well-defined loads. This may be accomplished by experimental testing techniques and/or by theoretical and mathematical stress analyses. These topics are treated in traditional stress analysis and strength of materials texts.

Materials and metallurgical engineers, on the other hand, are concerned with producing and fabricating materials to meet service requirements as predicted by these stress analyses. This necessarily involves an understanding of the relationships between the microstructure (i.e., internal features) of materials and their mechanical properties.

Materials are frequently chosen for structural applications because they have desirable combinations of mechanical characteristics. This chapter discusses the stress–strain behaviors of metals, ceramics, and polymers and the related mechanical properties; it also examines their other important mechanical characteristics. Discussions of the microscopic aspects of deformation mechanisms and methods to strengthen and regulate the mechanical behaviors are deferred to Chapter 8.

7.2 Concepts of Stress and Strain

If a load is static or changes relatively slowly with time and is applied uniformly over a cross section or surface of a member, the mechanical behavior may be ascertained by a simple stress–strain test; these are most commonly conducted for metals at room temperature. There are three principal ways in which a load may be applied: namely, tension, compression, and shear (Figures 7.1*a*, *b*, *c*). In engineering practice many loads are torsional rather than pure shear; this type of loading is illustrated in Figure 7.1*d*.

TENSION TESTS[1]

One of the most common mechanical stress–strain tests is performed in *tension*. As will be seen, the tension test can be used to ascertain several mechanical properties of materials that are important in design. A specimen is deformed, usually to fracture, with a gradually increasing tensile load that is applied uniaxially along the long axis of a specimen. A standard tensile specimen is shown in Figure 7.2. Normally, the cross section is circular, but rectangular specimens are also used. During testing, deformation is confined to the narrow center region, which has a uniform cross section along its length. The standard diameter is approximately 12.8 mm (0.5 in.), whereas the reduced section length should be at least four times this diameter; 60 mm ($2\frac{1}{4}$ in.) is common. Gauge length is used in ductility computations, as discussed in Section 7.6; the standard value is 50 mm (2.0 in.). The specimen is mounted by its ends into the holding grips of the testing apparatus (Figure 7.3). The tensile testing machine is designed to elongate the specimen at a constant rate, and to continuously and simultaneously measure the instantaneous applied load (with a load cell) and the resulting elongations (using an extensometer). A stress–strain test typically takes several minutes to perform and is destructive; that is, the test specimen is permanently deformed and usually fractured.

The output of such a tensile test is recorded on a strip chart (or by a computer) as load or force versus elongation. These load–deformation characteristics are dependent on the specimen size. For example, it will require twice the load to produce the same elongation if the cross-sectional area of the specimen is doubled. To minimize these geometrical factors, load and elongation are normalized to the respective parameters of **engineering stress** and **engineering strain.** Engineering

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[1] ASTM Standards E 8 and E 8M, ‘‘Standard Test Methods for Tension Testing of Metallic Materials.’’

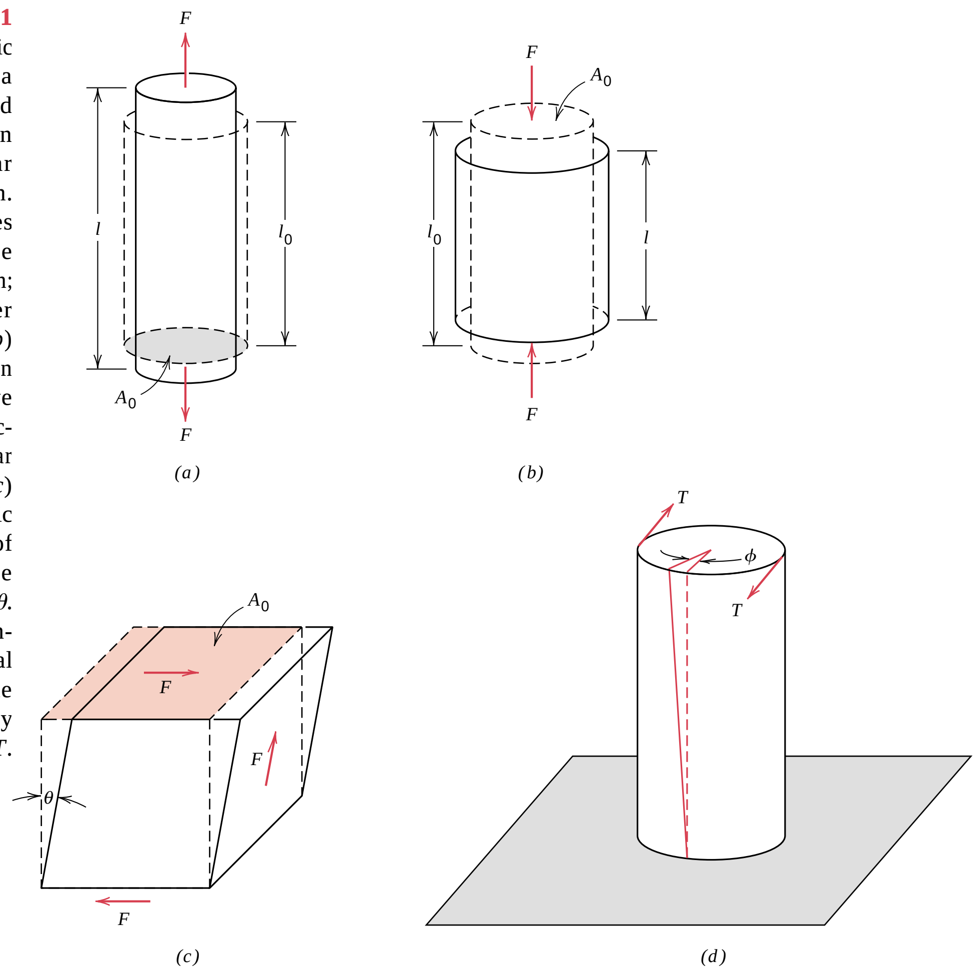


Figure 7.1 (*a*) Schematic illustration of how a tensile load produces an elongation and positive linear strain. Dashed lines represent the shape before deformation; solid lines, after deformation. (*b*) Schematic illustration of how a compressive load produces contraction and a negative linear strain. (*c*) Schematic representation of shear strain γ, where $\gamma = \tan \theta$. (*d*) Schematic representation of torsional deformation (i.e., angle of twist ϕ) produced by an applied torque T.

stress σ is defined by the relationship

$$\sigma = \frac{F}{A_0} \tag{7.1}$$

in which F is the instantaneous load applied perpendicular to the specimen cross section, in units of newtons (N) or pounds force (lb_f), and A_0 is the original cross-sectional area before any load is applied (m^2 or $in.^2$). The units of engineering

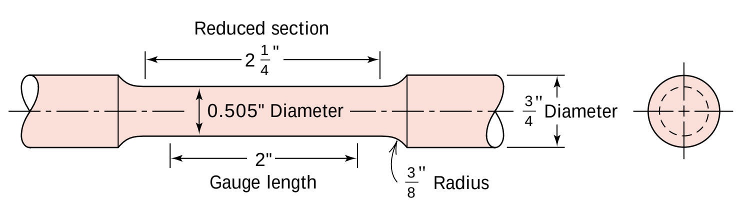


Figure 7.2 A standard tensile specimen with circular cross section.

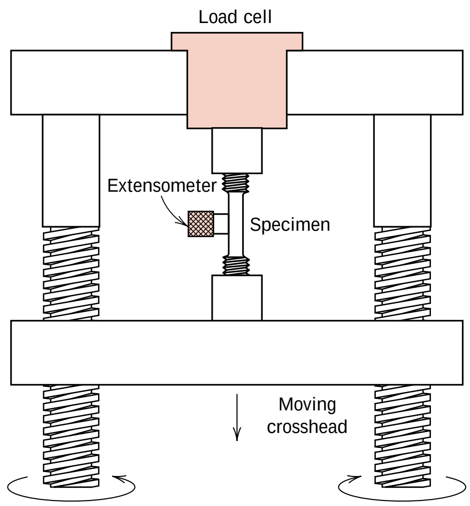


Figure 7.3 Schematic representation of the apparatus used to conduct tensile stress–strain tests. The specimen is elongated by the moving crosshead; load cell and extensometer measure, respectively, the magnitude of the applied load and the elongation. (Adapted from H. W. Hayden, W. G. Moffatt, and J. Wulff, *The Structure and Properties of Materials,* Vol. III, *Mechanical Behavior,* p. 2. Copyright © 1965 by John Wiley & Sons, New York. Reprinted by permission of John Wiley & Sons, Inc.)

stress (referred to subsequently as just stress) are megapascals, MPa (SI) (where 1 MPa $= 10^6$ N/m^2), and pounds force per square inch, psi (Customary U.S.).[2]

Engineering strain ϵ is defined according to

$$\epsilon = \frac{l_i - l_0}{l_0} = \frac{\Delta l}{l_0} \tag{7.2}$$

in which l_0 is the original length before any load is applied, and l_i is the instantaneous length. Sometimes the quantity $l_i - l_0$ is denoted as Δl, and is the deformation elongation or change in length at some instant, as referenced to the original length. Engineering strain (subsequently called just strain) is unitless, but meters per meter or inches per inch are often used; the value of strain is obviously independent of the unit system. Sometimes strain is also expressed as a percentage, in which the strain value is multiplied by 100.

COMPRESSION TESTS[3]

Compression stress–strain tests may be conducted if in-service forces are of this type. A compression test is conducted in a manner similar to the tensile test, except that the force is compressive and the specimen contracts along the direction of the stress. Equations 7.1 and 7.2 are utilized to compute compressive stress and strain, respectively. By convention, a compressive force is taken to be negative, which yields a negative stress. Furthermore, since l_0 is greater than l_i, compressive strains computed from Equation 7.2 are necessarily also negative. Tensile tests are more common because they are easier to perform; also, for most materials used in structural applications, very little additional information is obtained from compressive tests. Compressive tests are used when a material's behavior under large and perma-

[2] Conversion from one system of stress units to the other is accomplished by the relationship 145 psi = 1 MPa.

[3] ASTM Standard E 9, ''Standard Test Methods of Compression Testing of Metallic Materials at Room Temperature.''

nent (i.e., plastic) strains is desired, as in manufacturing applications, or when the material is brittle in tension.

SHEAR AND TORSIONAL TESTS[4]

For tests performed using a pure shear force as shown in Figure 7.1*c*, the shear stress τ is computed according to

$$\tau = \frac{F}{A_0} \tag{7.3}$$

where F is the load or force imposed parallel to the upper and lower faces, each of which has an area of A_0. The shear strain γ is defined as the tangent of the strain angle θ, as indicated in the figure. The units for shear stress and strain are the same as for their tensile counterparts.

Torsion is a variation of pure shear, wherein a structural member is twisted in the manner of Figure 7.1*d*; torsional forces produce a rotational motion about the longitudinal axis of one end of the member relative to the other end. Examples of torsion are found for machine axles and drive shafts, and also for twist drills. Torsional tests are normally performed on cylindrical solid shafts or tubes. A shear stress τ is a function of the applied torque T, whereas shear strain γ is related to the angle of twist, ϕ in Figure 7.1*d*.

GEOMETRIC CONSIDERATIONS OF THE STRESS STATE

Stresses that are computed from the tensile, compressive, shear, and torsional force states represented in Figure 7.1 act either parallel or perpendicular to planar faces of the bodies represented in these illustrations. It should be noted that the stress state is a function of the orientations of the planes upon which the stresses are taken to act. For example, consider the cylindrical tensile specimen of Figure 7.4

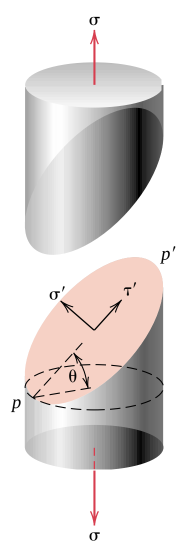


FIGURE 7.4 Schematic representation showing normal (σ') and shear (τ') stresses that act on a plane oriented at an angle θ relative to the plane taken perpendicular to the direction along which a pure tensile stress (σ) is applied.

[4] ASTM Standard E 143, ''Standard Test for Shear Modulus.''

that is subjected to a tensile stress σ applied parallel to its axis. Furthermore, consider also the plane p-p' that is oriented at some arbitrary angle θ relative to the plane of the specimen end-face. Upon this plane p-p', the applied stress is no longer a pure tensile one. Rather, a more complex stress state is present that consists of a tensile (or normal) stress σ' that acts normal to the p-p' plane, and, in addition, a shear stress τ' that acts parallel to this plane; both of these stresses are represented in the figure. Using mechanics of materials principles,[5] it is possible to develop equations for σ' and τ' in terms of σ and θ, as follows:

$$\sigma' = \sigma \cos^2 \theta = \sigma \left(\frac{1 + \cos 2\theta}{2} \right) \tag{7.4a}$$

$$\tau' = \sigma \sin \theta \cos \theta = \sigma \left(\frac{\sin 2\theta}{2} \right) \tag{7.4b}$$

These same mechanics principles allow the transformation of stress components from one coordinate system to another coordinate system that has a different orientation. Such treatments are beyond the scope of the present discussion.

ELASTIC DEFORMATION

7.3 Stress–Strain Behavior

The degree to which a structure deforms or strains depends on the magnitude of an imposed stress. For most metals that are stressed in tension and at relatively low levels, stress and strain are proportional to each other through the relationship

$$\sigma = E\epsilon \tag{7.5}$$

This is known as Hooke's law, and the constant of proportionality E (GPa or psi)[6] is the **modulus of elasticity,** or *Young's modulus.* For most typical metals the magnitude of this modulus ranges between 45 GPa (6.5×10^6 psi), for magnesium, and 407 GPa (59×10^6 psi), for tungsten. The moduli of elasticity are slightly higher for ceramic materials, which range between about 70 and 500 GPa (10×10^6 and 70×10^6 psi). Polymers have modulus values that are smaller than both metals and ceramics, and which lie in the range 0.007 and 4 GPa (10^3 and 0.6×10^6 psi). Room temperature modulus of elasticity values for a number of metals, ceramics, and polymers are presented in Table 7.1. A more comprehensive modulus list is provided in Table B.2, Appendix B.

Deformation in which stress and strain are proportional is called **elastic deformation;** a plot of stress (ordinate) versus strain (abscissa) results in a linear relationship, as shown in Figure 7.5. The slope of this linear segment corresponds to the modulus of elasticity E. This modulus may be thought of as stiffness, or a material's resistance to elastic deformation. The greater the modulus, the stiffer the material, or the smaller the elastic strain that results from the application of a given stress. The modulus is an important design parameter used for computing elastic deflections.

Elastic deformation is nonpermanent, which means that when the applied load is released, the piece returns to its original shape. As shown in the stress–strain

[5] See, for example, W. F. Riley, L. D. Sturges, and D. H. Morris, *Mechanics of Materials,* 5th edition, John Wiley & Sons, New York, 1999.

[6] The SI unit for the modulus of elasticity is gigapascal, GPa, where 1 GPa = 10^9 N/m^2 = 10^3 MPa.

Table 7.1 Room-Temperature Elastic and Shear Moduli, and Poisson's Ratio for Various Materials

Material	Modulus of Elasticity		Shear Modulus		Poisson's Ratio
	GPa	10^6 psi	GPa	10^6 psi	
Metal Alloys					
Tungsten	407	59	160	23.2	0.28
Steel	207	30	83	12.0	0.30
Nickel	207	30	76	11.0	0.31
Titanium	107	15.5	45	6.5	0.34
Copper	110	16	46	6.7	0.34
Brass	97	14	37	5.4	0.34
Aluminum	69	10	25	3.6	0.33
Magnesium	45	6.5	17	2.5	0.35
Ceramic Materials					
Aluminum oxide (Al_2O_3)	393	57	—	—	0.22
Silicon carbide (SiC)	345	50	—	—	0.17
Silicon nitride (Si_3N_4)	304	44	—	—	0.30
Spinel ($MgAl_2O_4$)	260	38	—	—	—
Magnesium oxide (MgO)	225	33	—	—	0.18
Zirconia[a]	205	30	—	—	0.31
Mullite ($3Al_2O_3$-$2SiO_2$)	145	21	—	—	0.24
Glass–ceramic (Pyroceram)	120	17	—	—	0.25
Fused silica (SiO_2)	73	11	—	—	0.17
Soda–lime glass	69	10	—	—	0.23
Polymers[b]					
Phenol-formaldehyde	2.76–4.83	0.40–0.70	—	—	—
Polyvinyl chloride (PVC)	2.41–4.14	0.35–0.60	—	—	0.38
Polyester (PET)	2.76–4.14	0.40–0.60	—	—	—
Polystyrene (PS)	2.28–3.28	0.33–0.48	—	—	0.33
Polymethyl methacrylate (PMMA)	2.24–3.24	0.33–0.47	—	—	—
Polycarbonate (PC)	2.38	0.35	—	—	0.36
Nylon 6,6	1.58–3.80	0.23–0.55	—	—	0.39
Polypropylene (PP)	1.14–1.55	0.17–0.23	—	—	—
Polyethylene—high density (HDPE)	1.08	0.16	—	—	—
Polytetrafluoroethylene (PTFE)	0.40–0.55	0.058–0.080	—	—	0.46
Polyethylene—low density (LDPE)	0.17–0.28	0.025–0.041	—	—	—

[a] Partially stabilized with 3 mol% Y_2O_3.

[b] **Source:** *Modern Plastics Encyclopedia '96.* Copyright 1995, The McGraw-Hill Companies. Reprinted with permission.

plot (Figure 7.5), application of the load corresponds to moving from the origin up and along the straight line. Upon release of the load, the line is traversed in the opposite direction, back to the origin.

There are some materials (e.g., gray cast iron, concrete, and many polymers) for which this initial elastic portion of the stress–strain curve is not linear (Figure

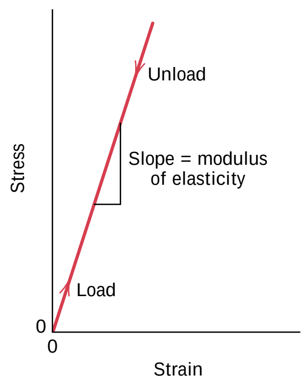


Figure 7.5 Schematic stress–strain diagram showing linear elastic deformation for loading and unloading cycles.

7.6); hence, it is not possible to determine a modulus of elasticity as described above. For this nonlinear behavior, either *tangent* or *secant modulus* is normally used. Tangent modulus is taken as the slope of the stress–strain curve at some specified level of stress, while secant modulus represents the slope of a secant drawn from the origin to some given point of the σ–ϵ curve. The determination of these moduli is illustrated in Figure 7.6.

On an atomic scale, macroscopic elastic strain is manifested as small changes in the interatomic spacing and the stretching of interatomic bonds. As a consequence, the magnitude of the modulus of elasticity is a measure of the resistance to separation of adjacent atoms/ions/molecules, that is, the interatomic bonding forces. Furthermore, this modulus is proportional to the slope of the interatomic force–separation curve (Figure 2.8*a*) at the equilibrium spacing:

$$E \propto \left(\frac{dF}{dr}\right)_{r_0} \tag{7.6}$$

Figure 7.7 shows the force–separation curves for materials having both strong and weak interatomic bonds; the slope at r_0 is indicated for each.

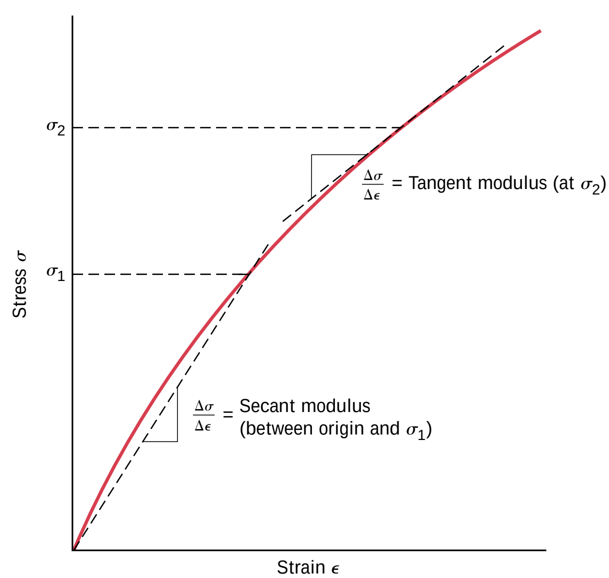


Figure 7.6 Schematic stress–strain diagram showing nonlinear elastic behavior, and how secant and tangent moduli are determined.

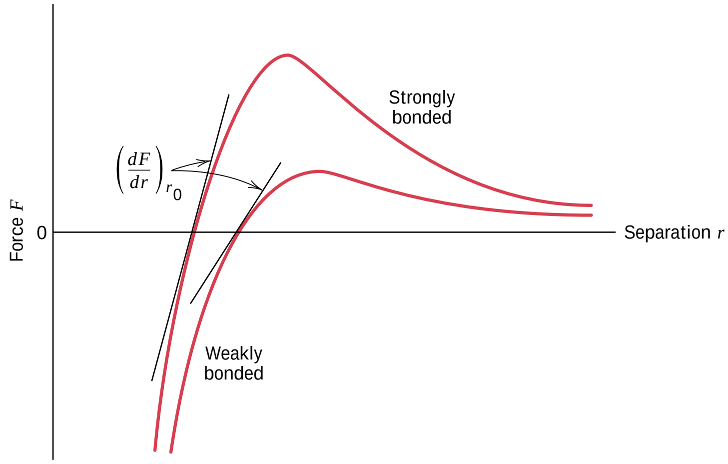


FIGURE 7.7 Force versus interatomic separation for weakly and strongly bonded atoms. The magnitude of the modulus of elasticity is proportional to the slope of each curve at the equilibrium interatomic separation r_0.

Differences in modulus values between metals, ceramics, and polymers are a direct consequence of the different types of atomic bonding that exist for the three materials types. Furthermore, with increasing temperature, the modulus of elasticity diminishes for all but some of the rubber materials; this effect is shown for several metals in Figure 7.8.

As would be expected, the imposition of compressive, shear, or torsional stresses also evokes elastic behavior. The stress–strain characteristics at low stress levels are virtually the same for both tensile and compressive situations, to include the magnitude of the modulus of elasticity. Shear stress and strain are proportional to each other through the expression

$$\tau = G\gamma \tag{7.7}$$

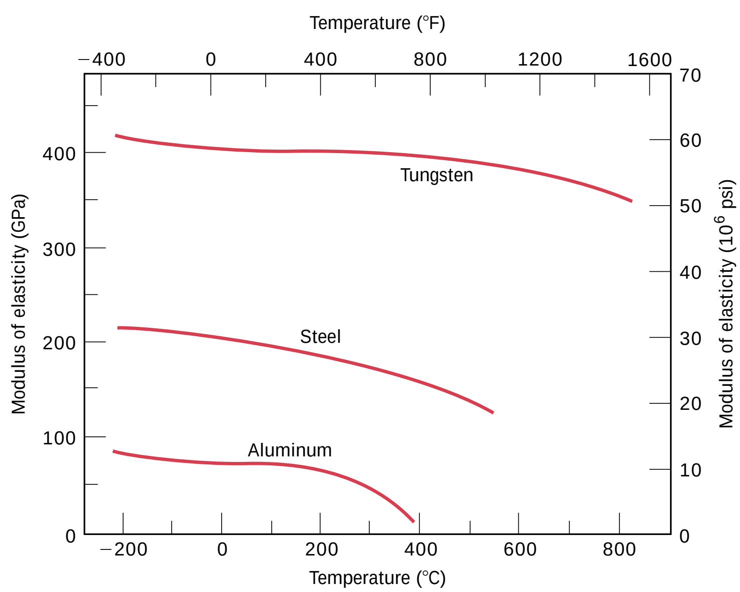


FIGURE 7.8 Plot of modulus of elasticity versus temperature for tungsten, steel, and aluminum. (Adapted from K. M. Ralls, T. H. Courtney, and J. Wulff, *Introduction to Materials Science and Engineering*. Copyright © 1976 by John Wiley & Sons, New York. Reprinted by permission of John Wiley & Sons, Inc.)

where G is the *shear modulus*, the slope of the linear elastic region of the shear stress–strain curve. Table 7.1 also gives the shear moduli for a number of the common metals.

7.4 Anelasticity

Up to this point, it has been assumed that elastic deformation is time independent, that is, that an applied stress produces an instantaneous elastic strain that remains constant over the period of time the stress is maintained. It has also been assumed that upon release of the load the strain is totally recovered, that is, that the strain immediately returns to zero. In most engineering materials, however, there will also exist a time-dependent elastic strain component. That is, elastic deformation will continue after the stress application, and upon load release some finite time is required for complete recovery. This time-dependent elastic behavior is known as **anelasticity,** and it is due to time-dependent microscopic and atomistic processes that are attendant to the deformation. For metals the anelastic component is normally small and is often neglected. However, for some polymeric materials its magnitude is significant; in this case it is termed *viscoelastic behavior,* {which is the discussion topic of Section 7.15.}

Example Problem 7.1

A piece of copper originally 305 mm (12 in.) long is pulled in tension with a stress of 276 MPa (40,000 psi). If the deformation is entirely elastic, what will be the resultant elongation?

Solution

Since the deformation is elastic, strain is dependent on stress according to Equation 7.5. Furthermore, the elongation Δl is related to the original length l_0 through Equation 7.2. Combining these two expressions and solving for Δl yields

$$\sigma = \epsilon E = \left(\frac{\Delta l}{l_0}\right) E$$

$$\Delta l = \frac{\sigma l_0}{E}$$

The values of σ and l_0 are given as 276 MPa and 305 mm, respectively, and the magnitude of E for copper from Table 7.1 is 110 GPa (16×10^6 psi). Elongation is obtained by substitution into the expression above as

$$\Delta l = \frac{(276 \text{ MPa})(305 \text{ mm})}{110 \times 10^3 \text{ MPa}} = 0.77 \text{ mm (0.03 in.)}$$

7.5 Elastic Properties of Materials

When a tensile stress is imposed on virtually all materials, an elastic elongation and accompanying strain ϵ_z result in the direction of the applied stress (arbitrarily taken to be the z direction), as indicated in Figure 7.9. As a result of this elongation,

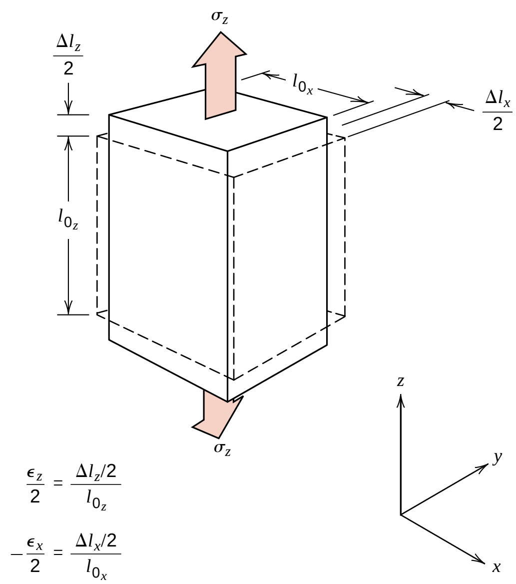


FIGURE 7.9 Axial (z) elongation (positive strain) and lateral (x and y) contractions (negative strains) in response to an imposed tensile stress. Solid lines represent dimensions after stress application; dashed lines, before.

there will be constrictions in the lateral (x and y) directions perpendicular to the applied stress; from these contractions, the compressive strains ϵ_x and ϵ_y may be determined. If the applied stress is uniaxial (only in the z direction), and the material is isotropic, then $\epsilon_x = \epsilon_y$. A parameter termed **Poisson's ratio** ν is defined as the ratio of the lateral and axial strains, or

$$\nu = -\frac{\epsilon_x}{\epsilon_z} = -\frac{\epsilon_y}{\epsilon_z} \tag{7.8}$$

The negative sign is included in the expression so that ν will always be positive, since ϵ_x and ϵ_z will always be of opposite sign. Theoretically, Poisson's ratio for isotropic materials should be $\frac{1}{4}$; furthermore, the maximum value for ν (or that value for which there is no net volume change) is 0.50. For many metals and other alloys, values of Poisson's ratio range between 0.25 and 0.35. Table 7.1 shows ν values for several common materials; a more comprehensive list is given in Table B.3, Appendix B.

For isotropic materials, shear and elastic moduli are related to each other and to Poisson's ratio according to

$$E = 2G(1 + \nu) \tag{7.9}$$

In most metals G is about $0.4E$; thus, if the value of one modulus is known, the other may be approximated.

Many materials are elastically anisotropic; that is, the elastic behavior (e.g., the magnitude of E) varies with crystallographic direction (see Table 3.7). For these materials the elastic properties are completely characterized only by the specification of several elastic constants, their number depending on characteristics of the crystal structure. Even for isotropic materials, for complete characterization of the elastic properties, at least two constants must be given. Since the grain orientation is random in most polycrystalline materials, these may be considered to be isotropic;

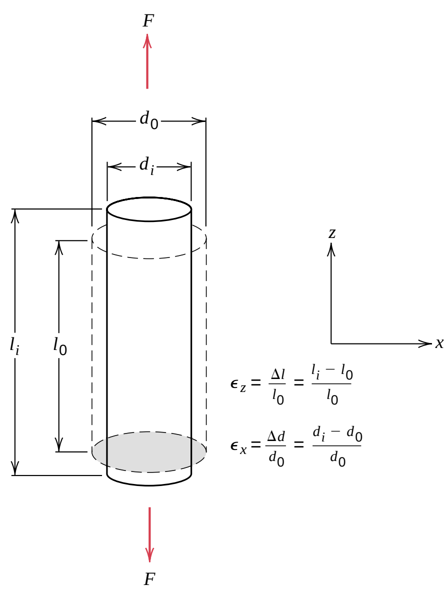
inorganic ceramic glasses are also isotropic. The remaining discussion of mechanical behavior assumes isotropy and polycrystallinity (for metals and crystalline ceramics) because such is the character of most engineering materials.

Example Problem 7.2

A tensile stress is to be applied along the long axis of a cylindrical brass rod that has a diameter of 10 mm (0.4 in.). Determine the magnitude of the load required to produce a 2.5×10^{-3} mm (10^{-4} in.) change in diameter if the deformation is entirely elastic.

Solution

This deformation situation is represented in the accompanying drawing.



When the force F is applied, the specimen will elongate in the z direction and at the same time experience a reduction in diameter, Δd, of 2.5×10^{-3} mm in the x direction. For the strain in the x direction,

$$\epsilon_x = \frac{\Delta d}{d_0} = \frac{-2.5 \times 10^{-3}\ \text{mm}}{10\ \text{mm}} = -2.5 \times 10^{-4}$$

which is negative, since the diameter is reduced.

It next becomes necessary to calculate the strain in the z direction using Equation 7.8. The value for Poisson's ratio for brass is 0.34 (Table 7.1), and thus

$$\epsilon_z = -\frac{\epsilon_x}{\nu} = -\frac{(-2.5 \times 10^{-4})}{0.34} = 7.35 \times 10^{-4}$$

The applied stress may now be computed using Equation 7.5 and the modulus of elasticity, given in Table 7.1 as 97 GPa (14×10^6 psi), as

$$\sigma = \epsilon_z E = (7.35 \times 10^{-4})(97 \times 10^3\ \text{MPa}) = 71.3\ \text{MPa}$$

Finally, from Equation 7.1, the applied force may be determined as

$$F = \sigma A_0 = \sigma \left(\frac{d_0}{2}\right)^2 \pi$$

$$= (71.3 \times 10^6 \text{ N/m}^2) \left(\frac{10 \times 10^{-3} \text{ m}}{2}\right)^2 \pi = 5600 \text{ N } (1293 \text{ lb}_f)$$

MECHANICAL BEHAVIOR—METALS

For most metallic materials, elastic deformation persists only to strains of about 0.005. As the material is deformed beyond this point, the stress is no longer proportional to strain (Hooke's law, Equation 7.5, ceases to be valid), and permanent, nonrecoverable, or **plastic deformation** occurs. Figure 7.10*a* plots schematically the tensile stress–strain behavior into the plastic region for a typical metal. The transition from elastic to plastic is a gradual one for most metals; some curvature results at the onset of plastic deformation, which increases more rapidly with rising stress.

From an atomic perspective, plastic deformation corresponds to the breaking of bonds with original atom neighbors and then reforming bonds with new neighbors as large numbers of atoms or molecules move relative to one another; upon removal of the stress they do not return to their original positions. This permanent deformation for metals is accomplished by means of a process called slip, which involves the motion of dislocations as discussed in Section 8.3.

7.6 Tensile Properties

YIELDING AND YIELD STRENGTH

Most structures are designed to ensure that only elastic deformation will result when a stress is applied. It is therefore desirable to know the stress level at which

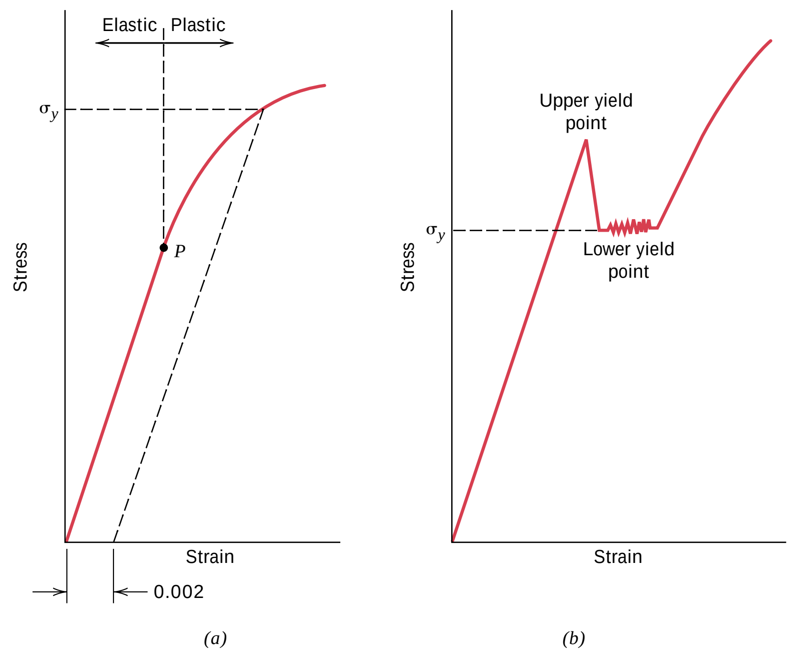


Figure 7.10 (*a*) Typical stress–strain behavior for a metal showing elastic and plastic deformations, the proportional limit *P*, and the yield strength σ_y, as determined using the 0.002 strain offset method. (*b*) Representative stress–strain behavior found for some steels demonstrating the yield point phenomenon.

plastic deformation begins, or where the phenomenon of **yielding** occurs. For metals that experience this gradual elastic–plastic transition, the point of yielding may be determined as the initial departure from linearity of the stress–strain curve; this is sometimes called the **proportional limit,** as indicated by point *P* in Figure 7.10*a*. In such cases the position of this point may not be determined precisely. As a consequence, a convention has been established wherein a straight line is constructed parallel to the elastic portion of the stress–strain curve at some specified strain offset, usually 0.002. The stress corresponding to the intersection of this line and the stress–strain curve as it bends over in the plastic region is defined as the **yield strength** σ_y.[7] This is demonstrated in Figure 7.10*a*. Of course, the units of yield strength are MPa or psi.[8]

For those materials having a nonlinear elastic region (Figure 7.6), use of the strain offset method is not possible, and the usual practice is to define the yield strength as the stress required to produce some amount of strain (e.g., $\epsilon = 0.005$).

Some steels and other materials exhibit the tensile stress–strain behavior as shown in Figure 7.10*b*. The elastic–plastic transition is very well defined and occurs abruptly in what is termed a *yield point phenomenon.* At the upper yield point, plastic deformation is initiated with an actual decrease in stress. Continued deformation fluctuates slightly about some constant stress value, termed the lower yield point; stress subsequently rises with increasing strain. For metals that display this effect, the yield strength is taken as the average stress that is associated with the lower yield point, since it is well defined and relatively insensitive to the testing procedure.[9] Thus, it is not necessary to employ the strain offset method for these materials.

The magnitude of the yield strength for a metal is a measure of its resistance to plastic deformation. Yield strengths may range from 35 MPa (5000 psi) for a low-strength aluminum to over 1400 MPa (200,000 psi) for high-strength steels.

TENSILE STRENGTH

After yielding, the stress necessary to continue plastic deformation in metals increases to a maximum, point *M* in Figure 7.11, and then decreases to the eventual fracture, point *F*. The **tensile strength** *TS* (MPa or psi) is the stress at the maximum on the engineering stress–strain curve (Figure 7.11). This corresponds to the maximum stress that can be sustained by a structure in tension; if this stress is applied and maintained, fracture will result. All deformation up to this point is uniform throughout the narrow region of the tensile specimen. However, at this maximum stress, a small constriction or neck begins to form at some point, and all subsequent deformation is confined at this neck, as indicated by the schematic specimen insets in Figure 7.11. This phenomenon is termed ''necking,'' and fracture ultimately occurs at the neck. The fracture strength corresponds to the stress at fracture.

Tensile strengths may vary anywhere from 50 MPa (7000 psi) for an aluminum to as high as 3000 MPa (450,000 psi) for the high-strength steels. Ordinarily, when the strength of a metal is cited for design purposes, the yield strength is used. This is because by the time a stress corresponding to the tensile strength has been

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[7] ''Strength'' is used in lieu of ''stress'' because strength is a property of the metal, whereas stress is related to the magnitude of the applied load.

[8] For Customary U.S. units, the unit of kilopounds per square inch (ksi) is sometimes used for the sake of convenience, where

$$1 \text{ ksi} = 1000 \text{ psi}$$

[9] It should be pointed out that to observe the yield point phenomenon, a ''stiff'' tensile-testing apparatus must be used; by stiff is meant that there is very little elastic deformation of the machine during loading.

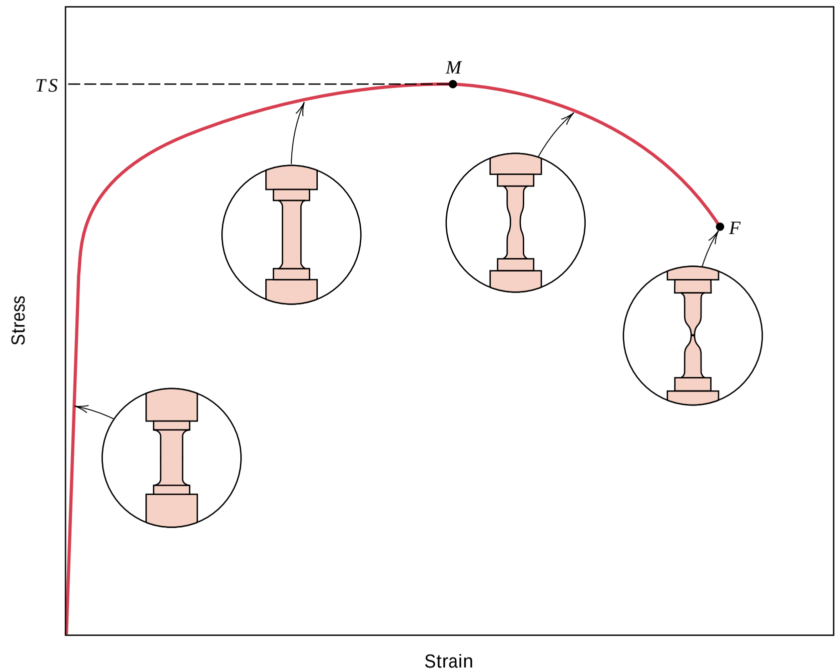

Figure 7.11 Typical engineering stress–strain behavior to fracture, point F. The tensile strength TS is indicated at point M. The circular insets represent the geometry of the deformed specimen at various points along the curve.

applied, often a structure has experienced so much plastic deformation that it is useless. Furthermore, fracture strengths are not normally specified for engineering design purposes.

Example Problem 7.3

From the tensile stress–strain behavior for the brass specimen shown in Figure 7.12, determine the following:

(a) The modulus of elasticity.

(b) The yield strength at a strain offset of 0.002.

(c) The maximum load that can be sustained by a cylindrical specimen having an original diameter of 12.8 mm (0.505 in.).

(d) The change in length of a specimen originally 250 mm (10 in.) long that is subjected to a tensile stress of 345 MPa (50,000 psi).

Solution

(a) The modulus of elasticity is the slope of the elastic or initial linear portion of the stress–strain curve. The strain axis has been expanded in the inset, Figure 7.12, to facilitate this computation. The slope of this linear region is the rise over the run, or the change in stress divided by the corresponding change in strain; in mathematical terms,

$$E = \text{slope} = \frac{\Delta\sigma}{\Delta\epsilon} = \frac{\sigma_2 - \sigma_1}{\epsilon_2 - \epsilon_1} \tag{7.10}$$

Inasmuch as the line segment passes through the origin, it is convenient to take both σ_1 and ϵ_1 as zero. If σ_2 is arbitrarily taken as 150 MPa, then ϵ_2 will have

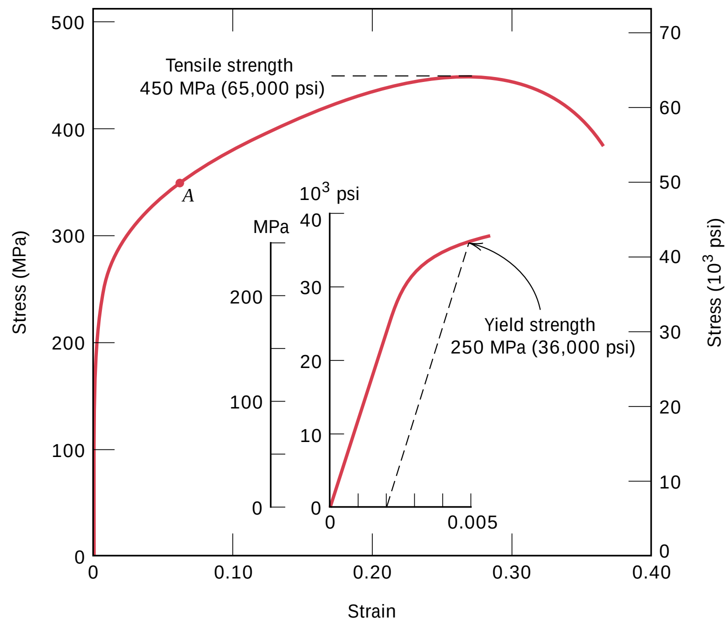

FIGURE 7.12 The stress–strain behavior for the brass specimen discussed in Example Problem 7.3.

a value of 0.0016. Therefore,

$$E = \frac{(150 - 0)\text{ MPa}}{0.0016 - 0} = 93.8\text{ GPa }(13.6 \times 10^6\text{ psi})$$

which is very close to the value of 97 GPa (14×10^6 psi) given for brass in Table 7.1.

(b) The 0.002 strain offset line is constructed as shown in the inset; its intersection with the stress–strain curve is at approximately 250 MPa (36,000 psi), which is the yield strength of the brass.

(c) The maximum load that can be sustained by the specimen is calculated by using Equation 7.1, in which σ is taken to be the tensile strength, from Figure 7.12, 450 MPa (65,000 psi). Solving for F, the maximum load, yields

$$F = \sigma A_0 = \sigma \left(\frac{d_0}{2}\right)^2 \pi$$

$$= (450 \times 10^6\text{ N/m}^2)\left(\frac{12.8 \times 10^{-3}\text{ m}}{2}\right)^2 \pi = 57{,}900\text{ N }(13{,}000\text{ lb}_f)$$

(d) To compute the change in length, Δl, in Equation 7.2, it is first necessary to determine the strain that is produced by a stress of 345 MPa. This is accomplished by locating the stress point on the stress–strain curve, point A, and reading the corresponding strain from the strain axis, which is approximately 0.06. Inasmuch as $l_0 = 250$ mm, we have

$$\Delta l = \epsilon l_0 = (0.06)(250\text{ mm}) = 15\text{ mm }(0.6\text{ in.})$$

DUCTILITY

Ductility is another important mechanical property. It is a measure of the degree of plastic deformation that has been sustained at fracture. A material that experiences very little or no plastic deformation upon fracture is termed *brittle.* The tensile stress–strain behaviors for both ductile and brittle materials are schematically illustrated in Figure 7.13.

Ductility may be expressed quantitatively as either *percent elongation* or *percent reduction in area.* The percent elongation %EL is the percentage of plastic strain at fracture, or

$$\%EL = \left(\frac{l_f - l_0}{l_0}\right) \times 100 \tag{7.11}$$

where l_f is the fracture length[10] and l_0 is the original gauge length as above. Inasmuch as a significant proportion of the plastic deformation at fracture is confined to the neck region, the magnitude of %EL will depend on specimen gauge length. The shorter l_0, the greater is the fraction of total elongation from the neck and, consequently, the higher the value of %EL. Therefore, l_0 should be specified when percent elongation values are cited; it is commonly 50 mm (2 in.).

Percent reduction in area %RA is defined as

$$\%RA = \left(\frac{A_0 - A_f}{A_0}\right) \times 100 \tag{7.12}$$

where A_0 is the original cross-sectional area and A_f is the cross-sectional area at the point of fracture.[10] Percent reduction in area values are independent of both l_0 and A_0. Furthermore, for a given material the magnitudes of %EL and %RA will, in general, be different. Most metals possess at least a moderate degree of ductility at room temperature; however, some become brittle as the temperature is lowered (Section 9.8).

A knowledge of the ductility of materials is important for at least two reasons. First, it indicates to a designer the degree to which a structure will deform plastically

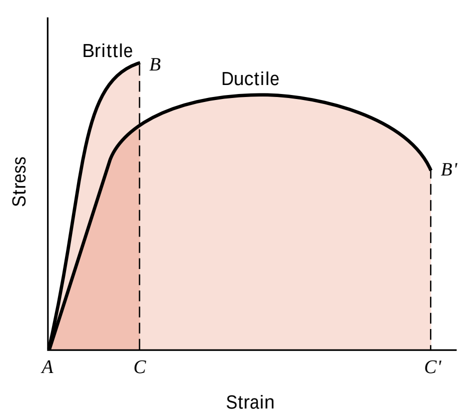

FIGURE 7.13 Schematic representations of tensile stress–strain behavior for brittle and ductile materials loaded to fracture.

[10] Both l_f and A_f are measured subsequent to fracture, and after the two broken ends have been repositioned back together.

before fracture. Second, it specifies the degree of allowable deformation during fabrication operations. We sometimes refer to relatively ductile materials as being ''forgiving,'' in the sense that they may experience local deformation without fracture should there be an error in the magnitude of the design stress calculation.

Brittle materials are *approximately* considered to be those having a fracture strain of less than about 5%.

Thus, several important mechanical properties of metals may be determined from tensile stress–strain tests. Table 7.2 presents some typical room-temperature

Table 7.2 Room-Temperature Mechanical Properties (in Tension) for Various Materials

	Yield Strength		*Tensile Strength*		
Material	***MPa***	***ksi***	***MPa***	***ksi***	***Ductility, %EL [in 50 mm (2 in.)]***[a]
			Metal Alloys[b]		
Molybdenum	565	82	655	95	35
Titanium	450	65	520	75	25
Steel (1020)	180	26	380	55	25
Nickel	138	20	480	70	40
Iron	130	19	262	38	45
Brass (70 Cu–30 Zn)	75	11	300	44	68
Copper	69	10	200	29	45
Aluminum	35	5	90	13	40
			Ceramic Materials[c]		
Zirconia (ZrO_2)[d]	—	—	800–1500	115–215	—
Silicon nitride (Si_3N_4)	—	—	250–1000	35–145	—
Aluminum oxide (Al_2O_3)	—	—	275–700	40–100	—
Silicon carbide (SiC)	—	—	100–820	15–120	—
Glass–ceramic (Pyroceram)	—	—	247	36	—
Mullite ($3Al_2O_3$-$2SiO_2$)	—	—	185	27	—
Spinel ($MgAl_2O_4$)	—	—	110–245	16–36	—
Fused silica (SiO_2)	—	—	110	16	—
Magnesium oxide (MgO)[e]	—	—	105	15	—
Soda–lime glass	—	—	69	10	—
			Polymers		
Nylon 6,6	44.8–82.8	6.5–12	75.9–94.5	11.0–13.7	15–300
Polycarbonate (PC)	62.1	9.0	62.8–72.4	9.1–10.5	110–150
Polyester (PET)	59.3	8.6	48.3–72.4	7.0–10.5	30–300
Polymethyl methacrylate (PMMA)	53.8–73.1	7.8–10.6	48.3–72.4	7.0–10.5	2.0–5.5
Polyvinyl chloride (PVC)	40.7–44.8	5.9–6.5	40.7–51.7	5.9–7.5	40–80
Phenol-formaldehyde	—	—	34.5–62.1	5.0–9.0	1.5–2.0
Polystyrene (PS)	—	—	35.9–51.7	5.2–7.5	1.2–2.5
Polypropylene (PP)	31.0–37.2	4.5–5.4	31.0–41.4	4.5–6.0	100–600
Polyethylene—high density (HDPE)	26.2–33.1	3.8–4.8	22.1–31.0	3.2–4.5	10–1200
Polytetrafluoroethylene (PTFE)	—	—	20.7–34.5	3.0–5.0	200–400
Polyethylene—low density (LDPE)	9.0–14.5	1.3–2.1	8.3–31.4	1.2–4.55	100–650

[a] For polymers, percent elongation at break.
[b] Property values are for metal alloys in an annealed state.
[c] The tensile strength of ceramic materials is taken as flexural strength (Section 7.10).
[d] Partially stabilized with 3 mol% Y_2O_3.
[e] Sintered and containing approximately 5% porosity.

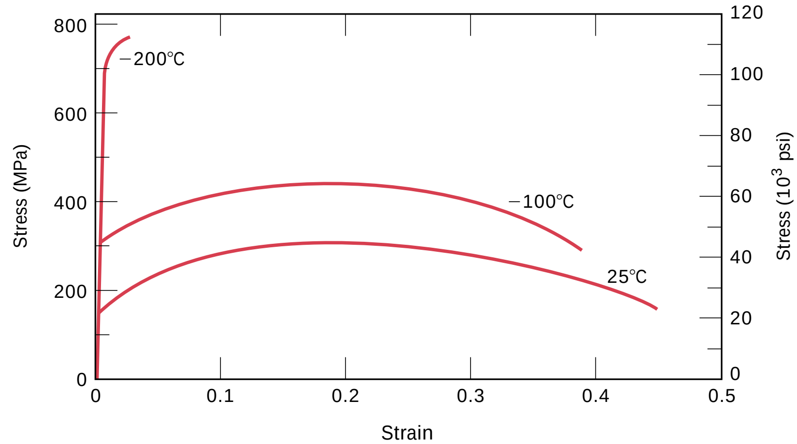

FIGURE 7.14 Engineering stress–strain behavior for iron at three temperatures.

values of yield strength, tensile strength, and ductility for several common metals (and also for a number of polymers and ceramics). These properties are sensitive to any prior deformation, the presence of impurities, and/or any heat treatment to which the metal has been subjected. The modulus of elasticity is one mechanical parameter that is insensitive to these treatments. As with modulus of elasticity, the magnitudes of both yield and tensile strengths decline with increasing temperature; just the reverse holds for ductility—it usually increases with temperature. Figure 7.14 shows how the stress–strain behavior of iron varies with temperature.

RESILIENCE

Resilience is the capacity of a material to absorb energy when it is deformed elastically and then, upon unloading, to have this energy recovered. The associated property is the *modulus of resilience*, U_r, which is the strain energy per unit volume required to stress a material from an unloaded state up to the point of yielding.

Computationally, the modulus of resilience for a specimen subjected to a uniaxial tension test is just the area under the engineering stress–strain curve taken to yielding (Figure 7.15), or

$$U_r = \int_0^{\epsilon_y} \sigma \, d\epsilon \tag{7.13a}$$

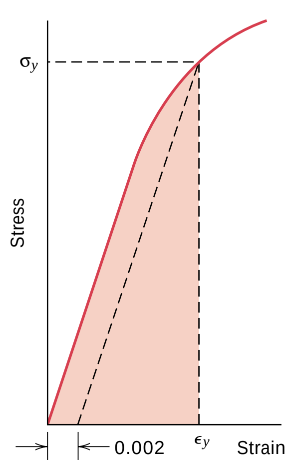

FIGURE 7.15 Schematic representation showing how modulus of resilience (corresponding to the shaded area) is determined from the tensile stress–strain behavior of a material.

Assuming a linear elastic region,

$$U_r = \tfrac{1}{2}\sigma_y\epsilon_y \tag{7.13b}$$

in which ϵ_y is the strain at yielding.

The units of resilience are the product of the units from each of the two axes of the stress–strain plot. For SI units, this is joules per cubic meter (J/m^3, equivalent to Pa), whereas with Customary U.S. units it is inch-pounds force per cubic inch (in.-lb$_f$/in.^{3}, equivalent to psi). Both joules and inch-pounds force are units of energy, and thus this area under the stress–strain curve represents energy absorption per unit volume (in cubic meters or cubic inches) of material.

Incorporation of Equation 7.5 into Equation 7.13b yields

$$U_r = \tfrac{1}{2}\sigma_y\epsilon_y = \tfrac{1}{2}\sigma_y\left(\frac{\sigma_y}{E}\right) = \frac{\sigma_y^2}{2E} \tag{7.14}$$

Thus, resilient materials are those having high yield strengths and low moduli of elasticity; such alloys would be used in spring applications.

TOUGHNESS

Toughness is a mechanical term that is used in several contexts; loosely speaking, it is a measure of the ability of a material to absorb energy up to fracture. Specimen geometry as well as the manner of load application are important in toughness determinations. For dynamic (high strain rate) loading conditions and when a notch (or point of stress concentration) is present, *notch toughness* is assessed by using an impact test, as discussed in Section 9.8. Furthermore, fracture toughness is a property indicative of a material's resistance to fracture when a crack is present (Section 9.5).

For the static (low strain rate) situation, toughness may be ascertained from the results of a tensile stress–strain test. It is the area under the σ–ϵ curve up to the point of fracture. The units for toughness are the same as for resilience (i.e., energy per unit volume of material). For a material to be tough, it must display both strength and ductility; and often, ductile materials are tougher than brittle ones. This is demonstrated in Figure 7.13, in which the stress–strain curves are plotted for both material types. Hence, even though the brittle material has higher yield and tensile strengths, it has a lower toughness than the ductile one, by virtue of lack of ductility; this is deduced by comparing the areas ABC and $AB'C'$ in Figure 7.13.

7.7 True Stress and Strain

From Figure 7.11, the decline in the stress necessary to continue deformation past the maximum, point M, seems to indicate that the metal is becoming weaker. This is not at all the case; as a matter of fact, it is increasing in strength. However, the cross-sectional area is decreasing rapidly within the neck region, where deformation is occurring. This results in a reduction in the load-bearing capacity of the specimen. The stress, as computed from Equation 7.1, is on the basis of the original cross-sectional area before any deformation, and does not take into account this diminution in area at the neck.

Sometimes it is more meaningful to use a true stress–true strain scheme. **True stress** σ_T is defined as the load F divided by the instantaneous cross-sectional area

A_i over which deformation is occurring (i.e., the neck, past the tensile point), or

$$\sigma_T = \frac{F}{A_i} \tag{7.15}$$

Furthermore, it is occasionally more convenient to represent strain as **true strain** ϵ_T, defined by

$$\epsilon_T = \ln \frac{l_i}{l_0} \tag{7.16}$$

If no volume change occurs during deformation, that is, if

$$A_i l_i = A_0 l_0 \tag{7.17}$$

true and engineering stress and strain are related according to

$$\sigma_T = \sigma(1 + \epsilon) \tag{7.18a}$$

$$\epsilon_T = \ln(1 + \epsilon) \tag{7.18b}$$

Equations 7.18a and 7.18b are valid only to the onset of necking; beyond this point true stress and strain should be computed from actual load, cross-sectional area, and gauge length measurements.

A schematic comparison of engineering and true stress–strain behavior is made in Figure 7.16. It is worth noting that the true stress necessary to sustain increasing strain continues to rise past the tensile point M'.

Coincident with the formation of a neck is the introduction of a complex stress state within the neck region (i.e., the existence of other stress components in addition to the axial stress). As a consequence, the correct stress (*axial*) within the neck is slightly lower than the stress computed from the applied load and neck cross-sectional area. This leads to the ''corrected'' curve in Figure 7.16.

For some metals and alloys the region of the true stress-strain curve from the onset of plastic deformation to the point at which necking begins may be approximated by

$$\sigma_T = K\epsilon_T^n \tag{7.19}$$

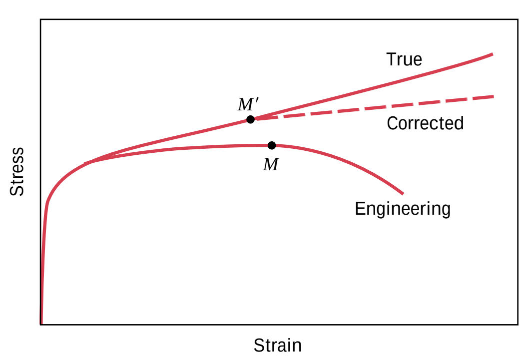

FIGURE 7.16 A comparison of typical tensile engineering stress–strain and true stress–strain behaviors. Necking begins at point M on the engineering curve, which corresponds to M' on the true curve. The ''corrected'' true stress–strain curve takes into account the complex stress state within the neck region.

Table 7.3 Tabulation of *n* and *K* Values (Equation 7.19) for Several Alloys

		K	
Material	***n***	***MPa***	***psi***
Low-carbon steel (annealed)	0.26	530	77,000
Alloy steel (Type 4340, annealed)	0.15	640	93,000
Stainless steel (Type 304, annealed)	0.45	1275	185,000
Aluminum (annealed)	0.20	180	26,000
Aluminum alloy (Type 2024, heat treated)	0.16	690	100,000
Copper (annealed)	0.54	315	46,000
Brass (70Cu–30Zn, annealed)	0.49	895	130,000

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In this expression, K and n are constants, which values will vary from alloy to alloy, and will also depend on the condition of the material (i.e., whether it has been plastically deformed, heat treated, etc.). The parameter n is often termed the *strain-hardening exponent* and has a value less than unity. Values of n and K for several alloys are contained in Table 7.3.

Example Problem 7.4

A cylindrical specimen of steel having an original diameter of 12.8 mm (0.505 in.) is tensile tested to fracture and found to have an engineering fracture strength σ_f of 460 MPa (67,000 psi). If its cross-sectional diameter at fracture is 10.7 mm (0.422 in.), determine:

(a) The ductility in terms of percent reduction in area.

(b) The true stress at fracture.

Solution

(a) Ductility is computed using Equation 7.12, as

$$\%\text{RA} = \frac{\left(\dfrac{12.8\text{ mm}}{2}\right)^2\pi - \left(\dfrac{10.7\text{ mm}}{2}\right)^2\pi}{\left(\dfrac{12.8\text{ mm}}{2}\right)^2\pi} \times 100$$

$$= \frac{128.7\text{ mm}^2 - 89.9\text{ mm}^2}{128.7\text{ mm}^2} \times 100 = 30\%$$

(b) True stress is defined by Equation 7.15, where in this case the area is taken as the fracture area A_f. However, the load at fracture must first be computed from the fracture strength as

$$F = \sigma_f A_0 = (460 \times 10^6\ \text{N/m}^2)(128.7\ \text{mm}^2)\left(\frac{1\ \text{m}^2}{10^6\ \text{mm}^2}\right) = 59{,}200\ \text{N}$$

Thus, the true stress is calculated as

$$\sigma_T = \frac{F}{A_f} = \frac{59{,}200\ \text{N}}{(89.9\ \text{mm}^2)\left(\frac{1\ \text{m}^2}{10^6\ \text{mm}^2}\right)}$$

$$= 6.6 \times 10^8\ \text{N/m}^2 = 660\ \text{MPa}\ (95{,}700\ \text{psi})$$

Example Problem 7.5

Compute the strain-hardening exponent n in Equation 7.19 for an alloy in which a true stress of 415 MPa (60,000 psi) produces a true strain of 0.10; assume a value of 1035 MPa (150,000 psi) for K.

Solution

This requires some algebraic manipulation of Equation 7.19 so that n becomes the dependent parameter. This is accomplished by taking logarithms and rearranging. Solving for n yields

$$n = \frac{\log \sigma_T - \log K}{\log \epsilon_T}$$

$$= \frac{\log(415\ \text{MPa}) - \log(1035\ \text{MPa})}{\log(0.1)} = 0.40$$

7.8 Elastic Recovery During Plastic Deformation

Upon release of the load during the course of a stress–strain test, some fraction of the total deformation is recovered as elastic strain. This behavior is demonstrated in Figure 7.17, a schematic engineering stress–strain plot. During the unloading cycle, the curve traces a near straight-line path from the point of unloading (point D), and its slope is virtually identical to the modulus of elasticity, or parallel to the initial elastic portion of the curve. The magnitude of this elastic strain, which is regained during unloading, corresponds to the strain recovery, as shown in Figure 7.17. If the load is reapplied, the curve will traverse essentially the same linear portion in the direction opposite to unloading; yielding will again occur at the unloading stress level where the unloading began. There will also be an elastic strain recovery associated with fracture.

7.9 Compressive, Shear, and Torsional Deformation

Of course, metals may experience plastic deformation under the influence of applied compressive, shear, and torsional loads. The resulting stress–strain behavior into

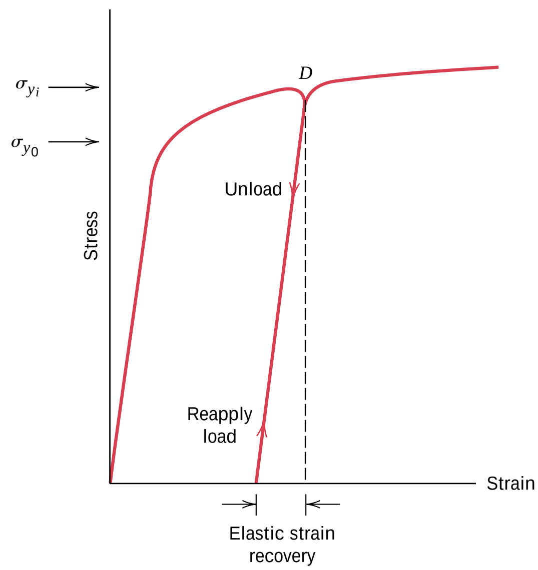


FIGURE 7.17 Schematic tensile stress–strain diagram showing the phenomena of elastic strain recovery and strain hardening. The initial yield strength is designated as σ_{y_0}; σ_{y_i} is the yield strength after releasing the load at point *D*, and then upon reloading.

the plastic region will be similar to the tensile counterpart (Figure 7.10*a*: yielding and the associated curvature). However, for compression, there will be no maximum, since necking does not occur; furthermore, the mode of fracture will be different from that for tension.

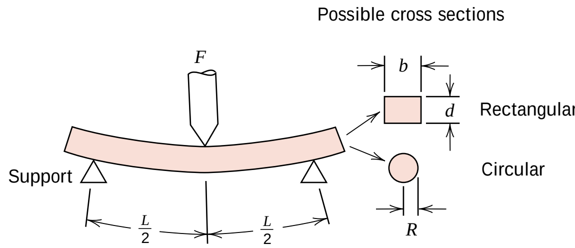
MECHANICAL BEHAVIOR—CERAMICS

Ceramic materials are somewhat limited in applicability by their mechanical properties, which in many respects are inferior to those of metals. The principal drawback is a disposition to catastrophic fracture in a brittle manner with very little energy absorption. In this section we explore the salient mechanical characteristics of these materials and how these properties are measured.

7.10 FLEXURAL STRENGTH

The stress–strain behavior of brittle ceramics is not usually ascertained by a tensile test as outlined in Section 7.2, for three reasons. First, it is difficult to prepare and test specimens having the required geometry. Second, it is difficult to grip brittle materials without fracturing them; and third, ceramics fail after only about 0.1% strain, which necessitates that tensile specimens be perfectly aligned in order to avoid the presence of bending stresses, which are not easily calculated. Therefore, a more suitable transverse bending test is most frequently employed, in which a rod specimen having either a circular or rectangular cross section is bent until fracture using a three- or four-point loading technique;[11] the three-point loading scheme is illustrated in Figure 7.18. At the point of loading, the top surface of the specimen is placed in a state of compression, whereas the bottom surface is in tension. Stress is computed from the specimen thickness, the bending moment, and

[11] ASTM Standard C 1161, ''Standard Test Method for Flexural Strength of Advanced Ceramics at Ambient Temperature.''



σ = stress = $\frac{Mc}{I}$

where M = maximum bending moment
c = distance from center of specimen to outer fibers
I = moment of inertia of cross section
F = applied load

	M	c	I	σ
Rectangular	$\frac{FL}{4}$	$\frac{d}{2}$	$\frac{bd^3}{12}$	$\frac{3FL}{2bd^2}$
Circular	$\frac{FL}{4}$	R	$\frac{\pi R^4}{4}$	$\frac{FL}{\pi R^3}$

FIGURE 7.18 A three-point loading scheme for measuring the stress–strain behavior and flexural strength of brittle ceramics, including expressions for computing stress for rectangular and circular cross sections.

the moment of inertia of the cross section; these parameters are noted in Figure 7.18 for rectangular and circular cross sections. The maximum tensile stress (as determined using these stress expressions) exists at the bottom specimen surface directly below the point of load application. Since the tensile strengths of ceramics are about one-tenth of their compressive strengths, and since fracture occurs on the tensile specimen face, the flexure test is a reasonable substitute for the tensile test.

The stress at fracture using this flexure test is known as the **flexural strength,** *modulus of rupture, fracture strength*, or the *bend strength*, an important mechanical parameter for brittle ceramics. For a rectangular cross section, the flexural strength σ_{fs} is equal to

$$\sigma_{fs} = \frac{3F_f L}{2bd^2} \tag{7.20a}$$

where F_f is the load at fracture, L is the distance between support points, and the other parameters are as indicated in Figure 7.18. When the cross section is circular, then

$$\sigma_{fs} = \frac{F_f L}{\pi R^3} \tag{7.20b}$$

R being the specimen radius.

Characteristic flexural strength values for several ceramic materials are given in Table 7.2. Since, during bending, a specimen is subjected to both compressive and tensile stresses, the magnitude of its flexural strength is greater than the tensile fracture strength. Furthermore, σ_{fs} will depend on specimen size; as explained in Section 9.6, with increasing specimen volume (under stress) there is an increase in the probability of the existence of a crack-producing flaw and, consequently, a decrease in flexural strength.

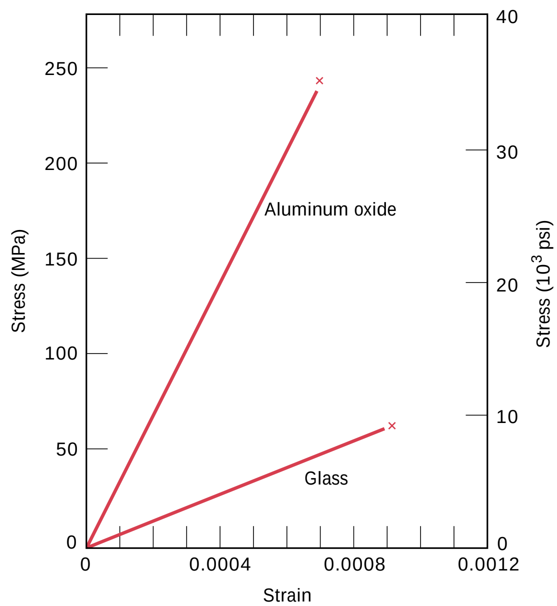


Figure 7.19 Typical stress–strain behavior to fracture for aluminum oxide and glass.

7.11 Elastic Behavior

The elastic stress–strain behavior for ceramic materials using these flexure tests is similar to the tensile test results for metals: a linear relationship exists between stress and strain. Figure 7.19 compares the stress–strain behavior to fracture for aluminum oxide (alumina) and glass. Again, the slope in the elastic region is the modulus of elasticity; also, the moduli of elasticity for ceramic materials are slightly higher than for metals (Table 7.2 and Table B.2, Appendix B). From Figure 7.19 it may be noted that neither of the materials experiences plastic deformation prior to fracture.

7.12 Influence of Porosity on the Mechanical Properties of Ceramics (CD-ROM)

MECHANICAL BEHAVIOR—POLYMERS

7.13 Stress–Strain Behavior

The mechanical properties of polymers are specified with many of the same parameters that are used for metals, that is, modulus of elasticity, and yield and tensile strengths. For many polymeric materials, the simple stress–strain test is employed for the characterization of some of these mechanical parameters.[12] The mechanical characteristics of polymers, for the most part, are highly sensitive to the rate of deformation (strain rate), the temperature, and the chemical nature of the environment (the presence of water, oxygen, organic solvents, etc.). Some modifications of the testing techniques and specimen configurations used for metals are necessary with polymers, especially for the highly elastic materials, such as rubbers.

[12] ASTM Standard D 638, ''Standard Test Method for Tensile Properties of Plastics.''

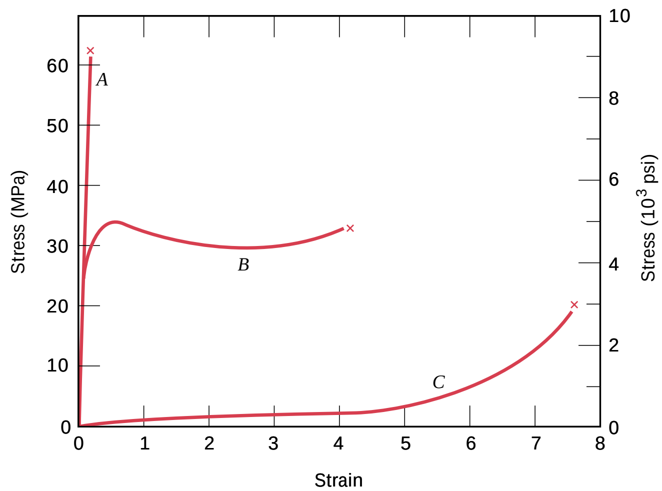


FIGURE 7.22 The stress–strain behavior for brittle (curve *A*), plastic (curve *B*), and highly elastic (elastomeric) (curve *C*) polymers.

Three typically different types of stress–strain behavior are found for polymeric materials, as represented in Figure 7.22. Curve *A* illustrates the stress–strain character for a brittle polymer, inasmuch as it fractures while deforming elastically. The behavior for the plastic material, curve *B*, is similar to that found for many metallic materials; the initial deformation is elastic, which is followed by yielding and a region of plastic deformation. Finally, the deformation displayed by curve *C* is totally elastic; this rubberlike elasticity (large recoverable strains produced at low stress levels) is displayed by a class of polymers termed the **elastomers.**

Modulus of elasticity (termed *tensile modulus* or sometimes just *modulus* for polymers) and ductility in percent elongation are determined for polymers in the same manner as for metals (Section 7.6). For plastic polymers (curve *B*, Figure 7.22), the yield point is taken as a maximum on the curve, which occurs just beyond the termination of the linear-elastic region (Figure 7.23); the stress at this maximum is the yield strength (σ_y). Furthermore, tensile strength (*TS*) corresponds to the stress at which fracture occurs (Figure 7.23); *TS* may be greater than or less than σ_y. Strength, for these plastic polymers, is normally taken as tensile strength. Table 7.2 and Tables B.2, B.3, and B.4 in Appendix B give these mechanical properties for a number of polymeric materials.

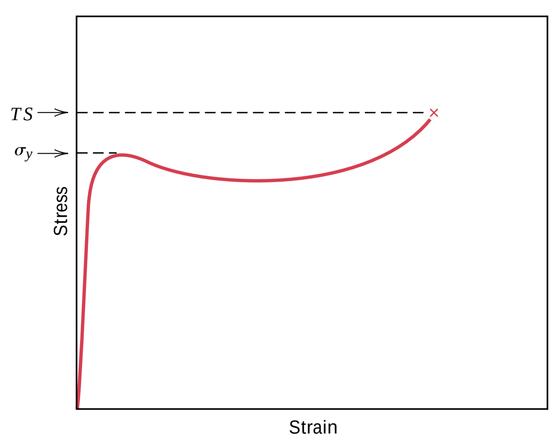


FIGURE 7.23 Schematic stress–strain curve for a plastic polymer showing how yield and tensile strengths are determined.

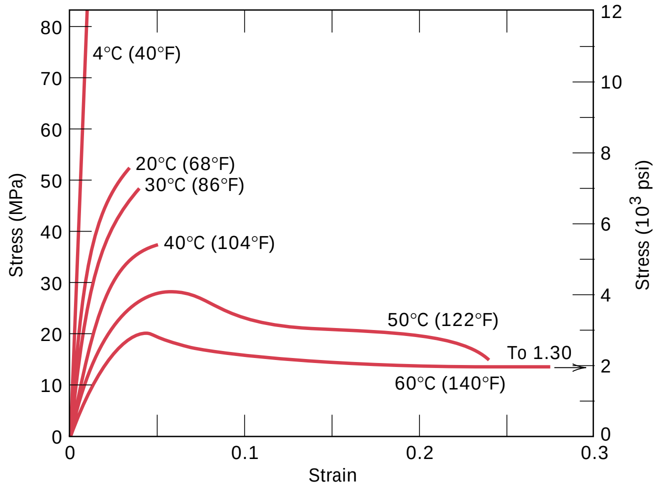


FIGURE 7.24 The influence of temperature on the stress–strain characteristics of polymethyl methacrylate. (From T. S. Carswell and H. K. Nason, ''Effect of Environmental Conditions on the Mechanical Properties of Organic Plastics,'' *Symposium on Plastics*, American Society for Testing and Materials, Philadelphia, 1944. Copyright, ASTM. Reprinted with permission.)

Polymers are, in many respects, mechanically dissimilar to metals (and ceramic materials). For example, the modulus for highly elastic polymeric materials may be as low as 7 MPa (10^3 psi), but may run as high as 4 GPa (0.6×10^6 psi) for some of the very stiff polymers; modulus values for metals are much larger (Table 7.1). Maximum tensile strengths for polymers are on the order of 100 MPa (15,000 psi)—for some metal alloys 4100 MPa (600,000 psi). And, whereas metals rarely elongate plastically to more than 100%, some highly elastic polymers may experience elongations to as much as 1000%.

In addition, the mechanical characteristics of polymers are much more sensitive to temperature changes within the vicinity of room temperature. Consider the stress–strain behavior for polymethyl methacrylate (Plexiglas) at several temperatures between 4 and 60°C (40 and 140°F) (Figure 7.24). Several features of this figure are worth noting, as follows: increasing the temperature produces (1) a decrease in elastic modulus, (2) a reduction in tensile strength, and (3) an enhancement of ductility—at 4°C (40°F) the material is totally brittle, whereas considerable plastic deformation is realized at both 50 and 60°C (122 and 140°F).

The influence of strain rate on the mechanical behavior may also be important. In general, decreasing the rate of deformation has the same influence on the stress–strain characteristics as increasing the temperature; that is, the material becomes softer and more ductile.

7.14 MACROSCOPIC DEFORMATION

Some aspects of the macroscopic deformation of semicrystalline polymers deserve our attention. The tensile stress–strain curve for a semicrystalline material, which was initially unoriented, is shown in Figure 7.25; also included in the figure are

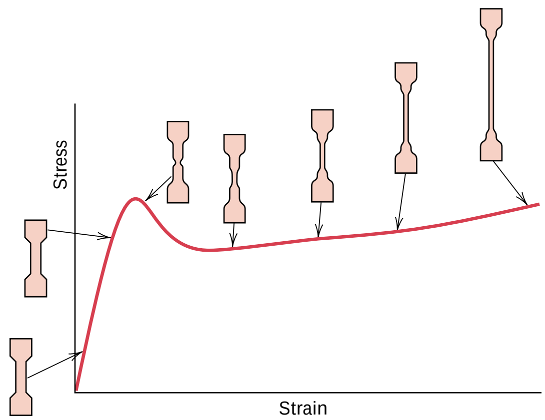


FIGURE 7.25 Schematic tensile stress–strain curve for a semicrystalline polymer. Specimen contours at several stages of deformation are included. (From Jerold M. Schultz, *Polymer Materials Science,* copyright © 1974, p. 488. Reprinted by permission of Prentice-Hall, Inc., Englewood Cliffs, NJ.)

schematic representations of specimen profile at various stages of deformation. Both upper and lower yield points are evident on the curve, which are followed by a near horizontal region. At the upper yield point, a small neck forms within the gauge section of the specimen. Within this neck, the chains become oriented (i.e., chain axes become aligned parallel to the elongation direction, a condition that is represented schematically in Figure 8.27*e*), which leads to localized strengthening. Consequently, there is a resistance to continued deformation at this point, and specimen elongation proceeds by the propagation of this neck region along the gauge length; the chain orientation phenomenon (Figure 8.27*e*) accompanies this neck extension. This tensile behavior may be contrasted to that found for ductile metals (Section 7.6), wherein once a neck has formed, all subsequent deformation is confined to within the neck region.

7.15 Viscoelasticity (CD-ROM)

HARDNESS AND OTHER MECHANICAL PROPERTY CONSIDERATIONS

7.16 Hardness

Another mechanical property that may be important to consider is **hardness,** which is a measure of a material's resistance to localized plastic deformation (e.g., a small dent or a scratch). Early hardness tests were based on natural minerals with a scale constructed solely on the ability of one material to scratch another that was softer. A qualitative and somewhat arbitrary hardness indexing scheme was devised, termed the Mohs scale, which ranged from 1 on the soft end for talc to 10 for diamond. Quantitative hardness techniques have been developed over the years in which a small indenter is forced into the surface of a material to be tested, under controlled conditions of load and rate of application. The depth or size of the resulting indentation is measured, which in turn is related to a hardness number; the softer the material, the larger and deeper the indentation, and the lower the hardness index number. Measured hardnesses are only relative (rather than absolute), and care should be exercised when comparing values determined by different techniques.

Hardness tests are performed more frequently than any other mechanical test for several reasons:

1. They are simple and inexpensive—ordinarily no special specimen need be prepared, and the testing apparatus is relatively inexpensive.
2. The test is nondestructive—the specimen is neither fractured nor excessively deformed; a small indentation is the only deformation.
3. Other mechanical properties often may be estimated from hardness data, such as tensile strength (see Figure 7.31).

ROCKWELL HARDNESS TESTS[13]

The Rockwell tests constitute the most common method used to measure hardness because they are so simple to perform and require no special skills. Several different scales may be utilized from possible combinations of various indenters and different loads, which permit the testing of virtually all metal alloys (as well as some polymers). Indenters include spherical and hardened steel balls having diameters of $\frac{1}{16}$, $\frac{1}{8}$, $\frac{1}{4}$, and $\frac{1}{2}$ in. (1.588, 3.175, 6.350, and 12.70 mm), and a conical diamond (Brale) indenter, which is used for the hardest materials.

With this system, a hardness number is determined by the difference in depth of penetration resulting from the application of an initial minor load followed by a larger major load; utilization of a minor load enhances test accuracy. On the basis of the magnitude of both major and minor loads, there are two types of tests: Rockwell and superficial Rockwell. For Rockwell, the minor load is 10 kg, whereas major loads are 60, 100, and 150 kg. Each scale is represented by a letter of the alphabet; several are listed with the corresponding indenter and load in Tables 7.4 and 7.5a. For superficial tests, 3 kg is the minor load; 15, 30, and 45 kg are the possible major load values. These scales are identified by a 15, 30, or 45 (according to load), followed by N, T, W, X, or Y, depending on indenter. Superficial tests are frequently performed on thin specimens. Table 7.5b presents several superficial scales.

When specifying Rockwell and superficial hardnesses, both hardness number and scale symbol must be indicated. The scale is designated by the symbol HR followed by the appropriate scale identification.[14] For example, 80 HRB represents a Rockwell hardness of 80 on the B scale, and 60 HR30W indicates a superficial hardness of 60 on the 30W scale.

For each scale, hardnesses may range up to 130; however, as hardness values rise above 100 or drop below 20 on any scale, they become inaccurate; and because the scales have some overlap, in such a situation it is best to utilize the next harder or softer scale.

Inaccuracies also result if the test specimen is too thin, if an indentation is made too near a specimen edge, or if two indentations are made too close to one another. Specimen thickness should be at least ten times the indentation depth, whereas allowance should be made for at least three indentation diameters between the center of one indentation and the specimen edge, or to the center of a second indentation. Furthermore, testing of specimens stacked one on top of another is not recommended. Also, accuracy is dependent on the indentation being made into a smooth flat surface.

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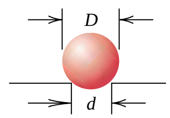
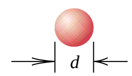
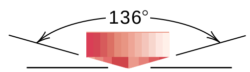
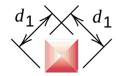
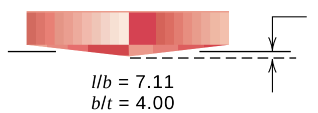
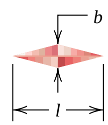
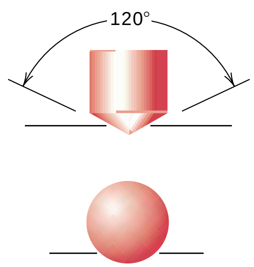
[13] ASTM Standard E 18, ''Standard Test Methods for Rockwell Hardness and Rockwell Superficial Hardness of Metallic Materials.''

[14] Rockwell scales are also frequently designated by an R with the appropriate scale letter as a subscript, for example, R_C denotes the Rockwell C scale.

Table 7.4 Hardness Testing Techniques

Test	***Indenter***	***Shape of Indentation***		***Load***	***Formula for Hardness Number***[a]
		Side View	***Top View***		
Brinell	10-mm sphere of steel or tungsten carbide			P	$\text{HB} = \dfrac{2P}{\pi D[D - \sqrt{D^2 - d^2}]}$
Vickers microhardness	Diamond pyramid			P	$\text{HV} = 1.854P/d_1^2$
Knoop microhardness	Diamond pyramid			P	$\text{HK} = 14.2P/l^2$
Rockwell and Superficial Rockwell	Diamond cone; $\frac{1}{16}$, $\frac{1}{8}$, $\frac{1}{4}$, $\frac{1}{2}$ in. diameter steel spheres			60 kg, 100 kg, 150 kg	Rockwell
				15 kg, 30 kg, 45 kg	Superficial Rockwell

[a] For the hardness formulas given, P (the applied load) is in kg, while D, d, d_1, and l are all in mm.

Source: Adapted from H. W. Hayden, W. G. Moffatt, and J. Wulff, *The Structure and Properties of Materials*, Vol. III, *Mechanical Behavior*. Copyright © 1965 by John Wiley & Sons, New York. Reprinted by permission of John Wiley & Sons, Inc.

Table 7.5a Rockwell Hardness Scales

Scale Symbol	*Indenter*	*Major Load (kg)*
A	Diamond	60
B	$\frac{1}{16}$ in. ball	100
C	Diamond	150
D	Diamond	100
E	$\frac{1}{8}$ in. ball	100
F	$\frac{1}{16}$ in. ball	60
G	$\frac{1}{16}$ in. ball	150
H	$\frac{1}{8}$ in. ball	60
K	$\frac{1}{8}$ in. ball	150

Table 7.5b Superficial Rockwell Hardness Scales

Scale Symbol	*Indenter*	*Major Load (kg)*
15N	Diamond	15
30N	Diamond	30
45N	Diamond	45
15T	$\frac{1}{16}$ in. ball	15
30T	$\frac{1}{16}$ in. ball	30
45T	$\frac{1}{16}$ in. ball	45
15W	$\frac{1}{8}$ in. ball	15
30W	$\frac{1}{8}$ in. ball	30
45W	$\frac{1}{8}$ in. ball	45

The modern apparatus for making Rockwell hardness measurements (see the chapter-opening photograph for this chapter) is automated and very simple to use; hardness is read directly, and each measurement requires only a few seconds.

The modern testing apparatus also permits a variation in the time of load application. This variable must also be considered in interpreting hardness data.

BRINELL HARDNESS TESTS[15]

In Brinell tests, as in Rockwell measurements, a hard, spherical indenter is forced into the surface of the metal to be tested. The diameter of the hardened steel (or tungsten carbide) indenter is 10.00 mm (0.394 in.). Standard loads range between 500 and 3000 kg in 500-kg increments; during a test, the load is maintained constant for a specified time (between 10 and 30 s). Harder materials require greater applied loads. The Brinell hardness number, HB, is a function of both the magnitude of the load and the diameter of the resulting indentation (see Table 7.4).[16] This diameter is measured with a special low-power microscope, utilizing a scale that is etched on the eyepiece. The measured diameter is then converted to the appropriate HB number using a chart; only one scale is employed with this technique.

Maximum specimen thickness as well as indentation position (relative to specimen edges) and minimum indentation spacing requirements are the same as for Rockwell tests. In addition, a well-defined indentation is required; this necessitates a smooth flat surface in which the indentation is made.

[15] ASTM Standard E 10, ''Standard Test Method for Brinell Hardness of Metallic Materials.''
[16] The Brinell hardness number is also represented by BHN.

KNOOP AND VICKERS MICROHARDNESS TESTS[17]

Two other hardness testing techniques are Knoop (pronounced *nūp*) and Vickers (sometimes also called diamond pyramid). For each test a very small diamond indenter having pyramidal geometry is forced into the surface of the specimen. Applied loads are much smaller than for Rockwell and Brinell, ranging between 1 and 1000 g. The resulting impression is observed under a microscope and measured; this measurement is then converted into a hardness number (Table 7.4). Careful specimen surface preparation (grinding and polishing) may be necessary to ensure a well-defined indentation that may be accurately measured. The Knoop and Vickers hardness numbers are designated by HK and HV, respectively,[18] and hardness scales for both techniques are approximately equivalent. Knoop and Vickers are referred to as microhardness testing methods on the basis of load and indenter size. Both are well suited for measuring the hardness of small, selected specimen regions; furthermore, Knoop is used for testing brittle materials such as ceramics.

There are other hardness-testing techniques that are frequently employed, but which will not be discussed here; these include ultrasonic microhardness, dynamic (Scleroscope), durometer (for plastic and elastomeric materials), and scratch hardness tests. These are described in references provided at the end of the chapter.

HARDNESS CONVERSION

The facility to convert the hardness measured on one scale to that of another is most desirable. However, since hardness is not a well-defined material property, and because of the experimental dissimilarities among the various techniques, a comprehensive conversion scheme has not been devised. Hardness conversion data have been determined experimentally and found to be dependent on material type and characteristics. The most reliable conversion data exist for steels, some of which are presented in Figure 7.30 for Knoop, Brinell, and two Rockwell scales; the Mohs scale is also included. Detailed conversion tables for various other metals and alloys are contained in ASTM Standard E 140, ''Standard Hardness Conversion Tables for Metals.'' In light of the preceding discussion, care should be exercised in extrapolation of conversion data from one alloy system to another.

CORRELATION BETWEEN HARDNESS AND TENSILE STRENGTH

Both tensile strength and hardness are indicators of a metal's resistance to plastic deformation. Consequently, they are roughly proportional, as shown in Figure 7.31, on page 182, for tensile strength as a function of the HB for cast iron, steel, and brass. The same proportionality relationship does not hold for all metals, as Figure 7.31 indicates. As a rule of thumb for most steels, the HB and the tensile strength are related according to

$$TS(\text{MPa}) = 3.45 \times \text{HB} \tag{7.25a}$$

$$TS(\text{psi}) = 500 \times \text{HB} \tag{7.25b}$$

---

[17] ASTM Standard E 92, ''Standard Test Method for Vickers Hardness of Metallic Materials,'' and ASTM Standard E 384, ''Standard Test for Microhardness of Materials.''

[18] Sometimes KHN and VHN are used to denote Knoop and Vickers hardness numbers, respectively.

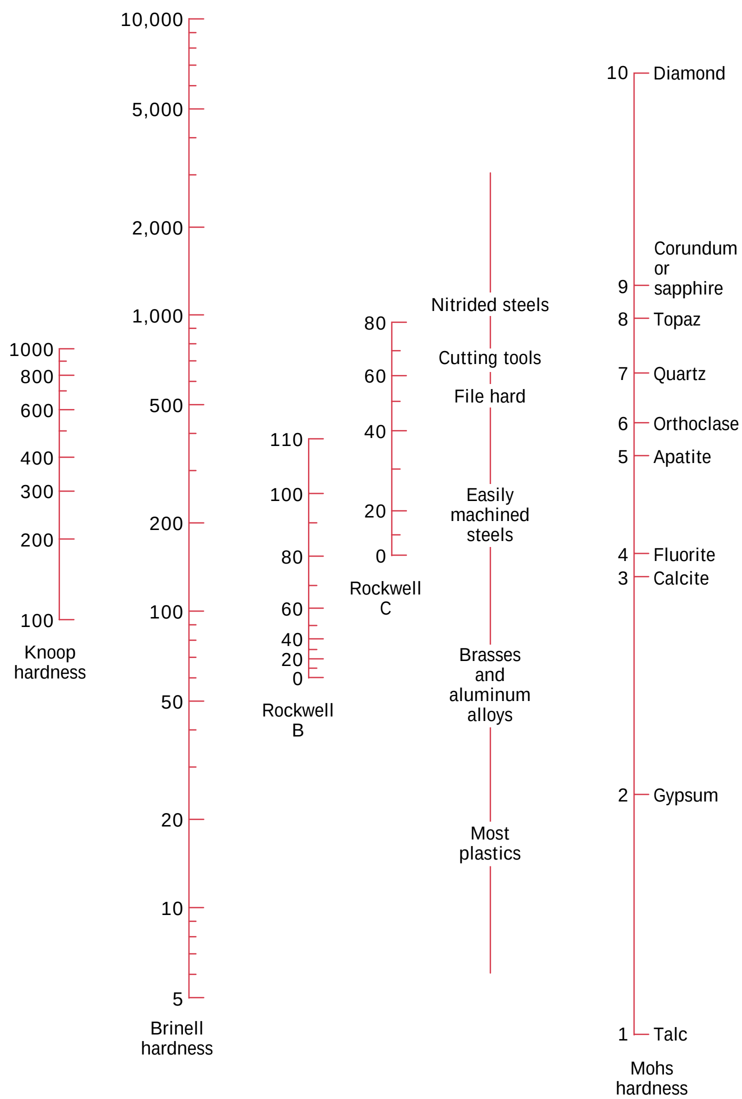


FIGURE 7.30 Comparison of several hardness scales. (Adapted from G. F. Kinney, *Engineering Properties and Applications of Plastics*, p. 202. Copyright © 1957 by John Wiley & Sons, New York. Reprinted by permission of John Wiley & Sons, Inc.)

7.17 HARDNESS OF CERAMIC MATERIALS

One beneficial mechanical property of ceramics is their hardness, which is often utilized when an abrasive or grinding action is required; in fact, the hardest known materials are ceramics. A listing of a number of different ceramic materials according to Knoop hardness is contained in Table 7.6. Only ceramics having Knoop hardnesses of about 1000 or greater are utilized for their abrasive characteristics (Section 13.8).

7.18 TEAR STRENGTH AND HARDNESS OF POLYMERS

Mechanical properties that are sometimes influential in the suitability of a polymer for some particular application include tear resistance and hardness. The ability to resist tearing is an important property of some plastics, especially those used for thin films in packaging. *Tear strength*, the mechanical parameter that is measured,

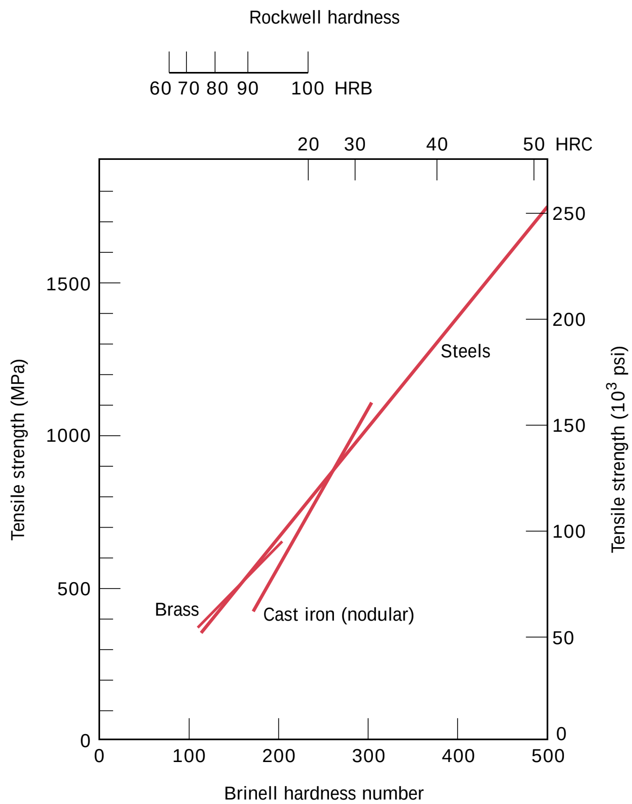


FIGURE 7.31 Relationships between hardness and tensile strength for steel, brass, and cast iron. (Data taken from *Metals Handbook: Properties and Selection: Irons and Steels,* Vol. 1, 9th edition, B. Bardes, Editor, American Society for Metals, 1978, pp. 36 and 461; and *Metals Handbook: Properties and Selection: Nonferrous Alloys and Pure Metals,* Vol. 2, 9th edition, H. Baker, Managing Editor, American Society for Metals, 1979, p. 327.)

is the energy required to tear apart a cut specimen that has a standard geometry. The magnitude of tensile and tear strengths are related.

Polymers are softer than metals and ceramics, and most hardness tests are conducted by penetration techniques similar to those described for metals in the previous section. Rockwell tests are frequently used for polymers.[19] Other indentation techniques employed are the Durometer and Barcol.[20]

Table 7.6 Approximate Knoop Hardness (100 g load) for Seven Ceramic Materials

Material	***Approximate Knoop Hardness***
Diamond (carbon)	7000
Boron carbide (B_4C)	2800
Silicon carbide (SiC)	2500
Tungsten carbide (WC)	2100
Aluminum oxide (Al_2O_3)	2100
Quartz (SiO_2)	800
Glass	550

[19] ASTM Standard D 785, ''Rockwell Hardness of Plastics and Electrical Insulating Materials.''

[20] ASTM Standard D 2240, ''Standard Test Method for Rubber Property—Durometer Hardness;'' and ASTM Standard D 2583, ''Standard Test Method for Indentation of Rigid Plastics by Means of a Barcol Impressor.''

PROPERTY VARIABILITY AND DESIGN/SAFETY FACTORS

7.19 Variability of Material Properties

At this point it is worthwhile to discuss an issue that sometimes proves troublesome to many engineering students, namely, that measured material properties are not exact quantities. That is, even if we have a most precise measuring apparatus and a highly controlled test procedure, there will always be some scatter or variability in the data that are collected from specimens of the same material. For example, consider a number of identical tensile samples that are prepared from a single bar of some metal alloy, which samples are subsequently stress–strain tested in the same apparatus. We would most likely observe that each resulting stress–strain plot is slightly different from the others. This would lead to a variety of modulus of elasticity, yield strength, and tensile strength values. A number of factors lead to uncertainties in measured data. These include the test method, variations in specimen fabrication procedures, operator bias, and apparatus calibration. Furthermore, inhomogeneities may exist within the same lot of material, and/or slight compositional and other differences from lot to lot. Of course, appropriate measures should be taken to minimize the possibility of measurement error, and also to mitigate those factors that lead to data variability.

It should also be mentioned that scatter exists for other measured material properties such as density, electrical conductivity, and coefficient of thermal expansion.

It is important for the design engineer to realize that scatter and variability of materials properties are inevitable and must be dealt with appropriately. On occasion, data must be subjected to statistical treatments and probabilities determined. For example, instead of asking the question, ''What is the fracture strength of this alloy?'' the engineer should become accustomed to asking the question, ''What is the probability of failure of this alloy under these given circumstances?''

It is often desirable to specify a typical value and degree of dispersion (or scatter) for some measured property; such is commonly accomplished by taking the average and the standard deviation, respectively.

COMPUTATION OF AVERAGE AND STANDARD DEVIATION VALUES (CD-ROM)

7.20 Design/Safety Factors

There will always be uncertainties in characterizing the magnitude of applied loads and their associated stress levels for in-service applications; ordinarily load calculations are only approximate. Furthermore, as noted in the previous section, virtually all engineering materials exhibit a variability in their measured mechanical properties. Consequently, design allowances must be made to protect against unanticipated failure. One way this may be accomplished is by establishing, for the particular application, a **design stress,** denoted as σ_d. For static situations and when ductile materials are used, σ_d is taken as the calculated stress level σ_c (on the basis of the estimated maximum load) multiplied by a *design factor, N′*, that is

$$\sigma_d = N'\sigma_c \tag{7.28}$$

where N' is greater than unity. Thus, the material to be used for the particular application is chosen so as to have a yield strength at least as high as this value of σ_d.

Alternatively, a **safe stress** or *working stress*, σ_w, is used instead of design stress. This safe stress is based on the yield strength of the material and is defined as the yield strength divided by a *factor of safety*, N, or

$$\sigma_w = \frac{\sigma_y}{N} \tag{7.29}$$

Utilization of design stress (Equation 7.28) is usually preferred since it is based on the anticipated maximum applied stress instead of the yield strength of the material; normally there is a greater uncertainty in estimating this stress level than in the specification of the yield strength. However, in the discussion of this text, we are concerned with factors that influence yield strengths, and not in the determination of applied stresses; therefore, the succeeding discussion will deal with working stresses and factors of safety.

The choice of an appropriate value of N is necessary. If N is too large, then component overdesign will result, that is, either too much material or a material having a higher-than-necessary strength will be used. Values normally range between 1.2 and 4.0. Selection of N will depend on a number of factors, including economics, previous experience, the accuracy with which mechanical forces and material properties may be determined, and, most important, the consequences of failure in terms of loss of life and/or property damage.

DESIGN EXAMPLE 7.1

A tensile-testing apparatus is to be constructed that must withstand a maximum load of 220,000 N (50,000 lb_f). The design calls for two cylindrical support posts, each of which is to support half of the maximum load. Furthermore, plain-carbon (1045) steel ground and polished shafting rounds are to be used; the minimum yield and tensile strengths of this alloy are 310 MPa (45,000 psi) and 565 MPa (82,000 psi), respectively. Specify a suitable diameter for these support posts.

SOLUTION

The first step in this design process is to decide on a factor safety, N, which then allows determination of a working stress according to Equation 7.29. In addition, to ensure that the apparatus will be safe to operate, we also want to minimize any elastic deflection of the rods during testing; therefore, a relatively conservative factor of safety is to be used, say $N = 5$. Thus, the working stress σ_w is just

$$\sigma_w = \frac{\sigma_y}{N}$$

$$= \frac{310 \text{ MPa}}{5} = 62 \text{ MPa (9000 psi)}$$

From the definition of stress, Equation 7.1,

$$A_0 = \left(\frac{d}{2}\right)^2 \pi = \frac{F}{\sigma_w}$$

where d is the rod diameter and F is the applied force; furthermore, each of the two rods must support half of the total force or 110,000 N (25,000 psi). Solving for

d leads to

$$d = 2\sqrt{\frac{F}{\pi\sigma_w}}$$

$$= 2\sqrt{\frac{110{,}000\ \text{N}}{\pi\,(62 \times 10^6\ \text{N/m}^2)}}$$

$$= 4.75 \times 10^{-2}\ \text{m} = 47.5\ \text{mm}\ (1.87\ \text{in.})$$

Therefore, the diameter of each of the two rods should be 47.5 mm or 1.87 in.

SUMMARY

A number of the important mechanical properties of materials have been discussed in this chapter. Concepts of stress and strain were first introduced. Stress is a measure of an applied mechanical load or force, normalized to take into account cross-sectional area. Two different stress parameters were defined—engineering stress and true stress. Strain represents the amount of deformation induced by a stress; both engineering and true strains are used.

Some of the mechanical characteristics of materials can be ascertained by simple stress–strain tests. There are four test types: tension, compression, torsion, and shear. Tensile are the most common. A material that is stressed first undergoes elastic, or nonpermanent, deformation, wherein stress and strain are proportional. The constant of proportionality is the modulus of elasticity for tension and compression, and is the shear modulus when the stress is shear. Poisson's ratio represents the negative ratio of transverse and longitudinal strains.

For metals, the phenomenon of yielding occurs at the onset of plastic or permanent deformation; yield strength is determined by a strain offset method from the stress–strain behavior, which is indicative of the stress at which plastic deformation begins. Tensile strength corresponds to the maximum tensile stress that may be sustained by a specimen, whereas percents elongation and reduction in area are measures of ductility—the amount of plastic deformation that has occurred at fracture. Resilience is the capacity of a material to absorb energy during elastic deformation; modulus of resilience is the area beneath the engineering stress–strain curve up to the yield point. Also, static toughness represents the energy absorbed during the fracture of a material, and is taken as the area under the entire engineering stress–strain curve. Ductile materials are normally tougher than brittle ones.

For the brittle ceramic materials, flexural strengths are determined by performing transverse bending tests to fracture. {Many ceramic bodies contain residual porosity, which is deleterious to both their moduli of elasticity and flexural strengths.}

On the basis of stress–strain behavior, polymers fall within three general classifications: brittle, plastic, and highly elastic. These materials are neither as strong nor as stiff as metals, and their mechanical properties are sensitive to changes in temperature and strain rate.

{Viscoelastic mechanical behavior, being intermediate between totally elastic and totally viscous, is displayed by a number of polymeric materials. It is characterized by the relaxation modulus, a time-dependent modulus of elasticity. The magnitude of the relaxation modulus is very sensitive to temperature; critical to the in-service temperature range for elastomers is this temperature dependence.}

Hardness is a measure of the resistance to localized plastic deformation. In several popular hardness-testing techniques (Rockwell, Brinell, Knoop, and Vickers) a small indenter is forced into the surface of the material, and an index number is determined on the basis of the size or depth of the resulting indentation. For many metals, hardness and tensile strength are approximately proportional to each other. In addition to their inherent brittleness, ceramic materials are distinctively hard. And polymers are relatively soft in comparison to the other material types.

Measured mechanical properties (as well as other material properties) are not exact and precise quantities, in that there will always be some scatter for the measured data. Typical material property values are commonly specified in terms of averages, whereas magnitudes of scatter may be expressed as standard deviations.

As a result of uncertainties in both measured mechanical properties and inservice applied stresses, safe or working stresses are normally utilized for design purposes. For ductile materials, safe stress is the ratio of the yield strength and a factor of safety.

IMPORTANT TERMS AND CONCEPTS

Anelasticity
Design stress
Ductility
Elastic deformation
Elastic recovery
Elastomer
Engineering strain
Engineering stress
Flexural strength
Hardness
Modulus of elasticity
Plastic deformation
Poisson's ratio
Proportional limit
{Relaxation modulus}
Resilience
Safe stress
Shear
Tensile strength
Toughness
True strain
True stress
{Viscoelasticity}
Yielding
Yield strength

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QUESTIONS AND PROBLEMS

Note: To solve those problems having an asterisk (*) by their numbers, consultation of supplementary topics [appearing only on the CD-ROM (and not in print)] will probably be necessary.

7.1 Using mechanics of materials principles (i.e., equations of mechanical equilibrium applied to a free-body diagram), derive Equations 7.4a and 7.4b.

7.2 **(a)** Equations 7.4a and 7.4b are expressions for normal (σ') and shear (τ') stresses, respectively, as a function of the applied tensile stress (σ) and the inclination angle of the plane on which these stresses are taken (θ of Figure 7.4). Make a plot on which is presented the orientation parameters of these expressions (i.e., $\cos^2\theta$ and $\sin\theta\cos\theta$) versus θ.

(b) From this plot, at what angle of inclination is the normal stress a maximum?

(c) Also, at what inclination angle is the shear stress a maximum?

7.3 A specimen of aluminum having a rectangular cross section 10 mm × 12.7 mm (0.4 in. × 0.5 in.) is pulled in tension with 35,500 N (8000 lb_f) force, producing only elastic deformation. Calculate the resulting strain.

7.4 A cylindrical specimen of a titanium alloy having an elastic modulus of 107 GPa (15.5×10^6 psi) and an original diameter of 3.8 mm (0.15 in.) will experience only elastic deformation when a tensile load of 2000 N (450 lb_f) is applied. Compute the maximum length of the specimen before deformation if the maximum allowable elongation is 0.42 mm (0.0165 in.).

7.5 A steel bar 100 mm (4.0 in.) long and having a square cross section 20 mm (0.8 in.) on an edge is pulled in tension with a load of 89,000 N (20,000 lb_f), and experiences an elongation of 0.10 mm (4.0×10^{-3} in.). Assuming that the deformation is entirely elastic, calculate the elastic modulus of the steel.

7.6 Consider a cylindrical titanium wire 3.0 mm (0.12 in.) in diameter and 2.5×10^4 mm (1000 in.) long. Calculate its elongation when a load of 500 N (112 lb_f) is applied. Assume that the deformation is totally elastic.

7.7 For a bronze alloy, the stress at which plastic deformation begins is 275 MPa (40,000 psi), and the modulus of elasticity is 115 GPa (16.7×10^6 psi).

(a) What is the maximum load that may be applied to a specimen with a cross-sectional area of 325 mm^2 (0.5 $in.^2$) without plastic deformation?

(b) If the original specimen length is 115 mm (4.5 in.), what is the maximum length to which it may be stretched without causing plastic deformation?

7.8 A cylindrical rod of copper (E = 110 GPa,

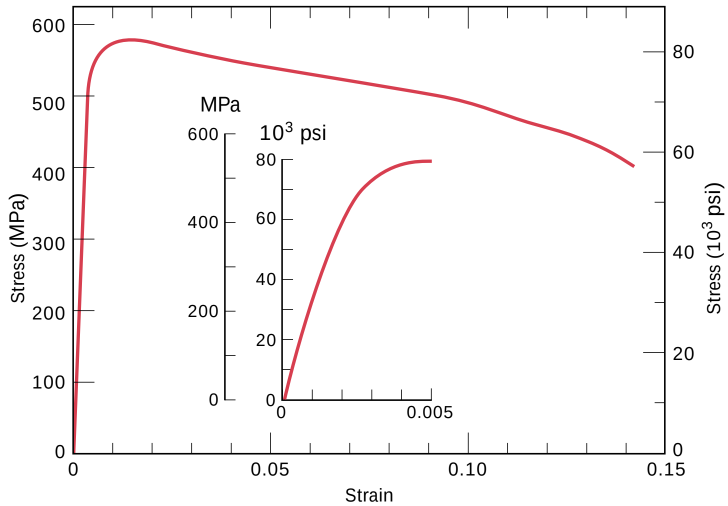

Figure 7.33 Tensile stress–strain behavior for a plain carbon steel.

16×10^6 psi) having a yield strength of 240 MPa (35,000 psi) is to be subjected to a load of 6660 N (1500 lb_f). If the length of the rod is 380 mm (15.0 in.), what must be the diameter to allow an elongation of 0.50 mm (0.020 in.)?

7.9 Consider a cylindrical specimen of a steel alloy (Figure 7.33) 10 mm (0.39 in.) in diameter and 75 mm (3.0 in.) long that is pulled in tension. Determine its elongation when a load of 23,500 N (5300 lb_f) is applied.

7.10 Figure 7.34 shows, for a gray cast iron, the tensile engineering stress–strain curve in the elastic region. Determine (**a**) the secant modulus taken to 35 MPa (5000 psi), and (**b**) the tangent modulus taken from the origin.

7.11 As was noted in Section 3.18, for single crystals of some substances, the physical properties are anisotropic, that is, they are dependent on crystallographic direction. One such property is the modulus of elasticity. For cubic single crystals, the modulus of elasticity in a general $[uvw]$ direction, E_{uvw}, is described by the relationship

$$\frac{1}{E_{uvw}} = \frac{1}{E_{\langle 100\rangle}} - 3\left(\frac{1}{E_{\langle 100\rangle}} - \frac{1}{E_{\langle 111\rangle}}\right)(\alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2)$$

where $E_{\langle 100\rangle}$ and $E_{\langle 111\rangle}$ are the moduli of elasticity in [100] and [111] directions, respectively; α, β, and γ are the cosines of the

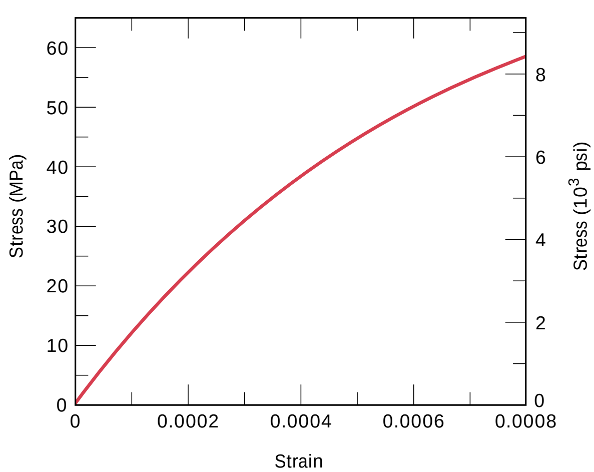

Figure 7.34 Tensile stress–strain behavior for a gray cast iron.

angles between $[uvw]$ and the respective [100], [010], and [001] directions. Verify that the $E_{\langle 110 \rangle}$ values for aluminum, copper, and iron in Table 3.7 are correct.

7.12 In Section 2.6 it was noted that the net bonding energy E_N between two isolated positive and negative ions is a function of interionic distance r as follows:

$$E_N = -\frac{A}{r} + \frac{B}{r^n} \qquad (7.30)$$

where A, B, and n are constants for the particular ion pair. Equation 7.30 is also valid for the bonding energy between adjacent ions in solid materials. The modulus of elasticity E is proportional to the slope of the interionic force-separation curve at the equilibrium interionic separation; that is,

$$E \propto \left(\frac{dF}{dr}\right)_{r_0}$$

Derive an expression for the dependence of the modulus of elasticity on these A, B, and n parameters (for the two-ion system) using the following procedure:

1. Establish a relationship for the force F as a function of r, realizing that

$$F = \frac{dE_N}{dr}$$

2. Now take the derivative dF/dr.

3. Develop an expression for r_0, the equilibrium separation. Since r_0 corresponds to the value of r at the minimum of the E_N-versus-r-curve (Figure 2.8*b*), take the derivative dE_N/dr, set it equal to zero, and solve for r, which corresponds to r_0.

4. Finally, substitute this expression for r_0 into the relationship obtained by taking dF/dr.

7.13 Using the solution to Problem 7.12, rank the magnitudes of the moduli of elasticity for the following hypothetical X, Y, and Z materials from the greatest to the least. The appropriate A, B, and n parameters (Equation 7.30) for these three materials are tabulated below; they yield E_N in units of electron volts and r in nanometers:

Material	*A*	*B*	*n*
X	2.5	2×10^{-5}	8
Y	2.3	8×10^{-6}	10.5
Z	3.0	1.5×10^{-5}	9

7.14 A cylindrical specimen of aluminum having a diameter of 19 mm (0.75 in.) and length of 200 mm (8.0 in.) is deformed elastically in tension with a force of 48,800 N (11,000 lb_f). Using the data contained in Table 7.1, determine the following:

(a) The amount by which this specimen will elongate in the direction of the applied stress.

(b) The change in diameter of the specimen. Will the diameter increase or decrease?

7.15 A cylindrical bar of steel 10 mm (0.4 in.) in diameter is to be deformed elastically by application of a force along the bar axis. Using the data in Table 7.1, determine the force that will produce an elastic reduction of 3×10^{-3} mm (1.2×10^{-4} in.) in the diameter.

7.16 A cylindrical specimen of some alloy 8 mm (0.31 in.) in diameter is stressed elastically in tension. A force of 15,700 N (3530 lb_f) produces a reduction in specimen diameter of 5×10^{-3} mm (2×10^{-4} in.). Compute Poisson's ratio for this material if its modulus of elasticity is 140 GPa (20.3×10^6 psi).

7.17 A cylindrical specimen of a hypothetical metal alloy is stressed in compression. If its original and final diameters are 20.000 and 20.025 mm, respectively, and its final length is 74.96 mm, compute its original length if the deformation is totally elastic. The elastic and shear moduli for this alloy are 105 GPa and 39.7 GPa, respectively.

7.18 Consider a cylindrical specimen of some hypothetical metal alloy that has a diameter of 8.0 mm (0.31 in.). A tensile force of 1000 N (225 lb_f) produces an elastic reduction in diameter of 2.8×10^{-4} mm (1.10×10^{-5} in.). Compute the modulus of elasticity for this alloy, given that Poisson's ratio is 0.30.

7.19 A brass alloy is known to have a yield strength of 275 MPa (40,000 psi), a tensile strength of 380 MPa (55,000 psi), and an elastic modulus of 103 GPa (15.0×10^6 psi). A cylindrical specimen of this alloy 12.7 mm (0.50 in.) in diameter and 250 mm (10.0 in.) long is stressed in tension and found to elongate 7.6 mm (0.30 in.). On the basis of the information given, is it possible to compute the magnitude of the load that is necessary to produce this change in length? If so, calculate the load. If not, explain why.

7.20 A cylindrical metal specimen 15.0 mm (0.59 in.) in diameter and 150 mm (5.9 in.) long is to be subjected to a tensile stress of 50 MPa (7250 psi); at this stress level the resulting deformation will be totally elastic.

(a) If the elongation must be less than 0.072 mm (2.83×10^{-3} in.), which of the metals in Table 7.1 are suitable candidates? Why?

(b) If, in addition, the maximum permissible diameter decrease is 2.3×10^{-3} mm (9.1×10^{-5} in.), which of the metals in Table 7.1 may be used? Why?

7.21 Consider the brass alloy with stress–strain behavior shown in Figure 7.12. A cylindrical specimen of this material 6 mm (0.24 in.) in diameter and 50 mm (2 in.) long is pulled in tension with a force of 5000 N (1125 lb_f). If it is known that this alloy has a Poisson's ratio of 0.30, compute: **(a)** the specimen elongation, and **(b)** the reduction in specimen diameter.

7.22 Cite the primary differences between elastic, anelastic, and plastic deformation behaviors.

7.23 A cylindrical rod 100 mm long and having a diameter of 10.0 mm is to be deformed using a tensile load of 27,500 N. It must not experience either plastic deformation or a diameter reduction of more than 7.5×10^{-3} mm. Of the materials listed as follows, which are possible candidates? Justify your choice(s).

Material	*Modulus of Elasticity* (*GPa*)	*Yield Strength* (*MPa*)	*Poisson's Ratio*
Aluminum alloy	70	200	0.33
Brass alloy	101	300	0.35
Steel alloy	207	400	0.27
Titanium alloy	107	650	0.36

7.24 A cylindrical rod 380 mm (15.0 in.) long, having a diameter of 10.0 mm (0.40 in.), is to be subjected to a tensile load. If the rod is to experience neither plastic deformation nor an elongation of more than 0.9 mm (0.035 in.) when the applied load is 24,500 N (5500 lb_f), which of the four metals or alloys listed below are possible candidates? Justify your choice(s).

Material	*Modulus of Elasticity* (*GPa*)	*Yield Strength* (*MPa*)	*Tensile Strength* (*MPa*)
Aluminum alloy	70	255	420
Brass alloy	100	345	420
Copper	110	250	290
Steel alloy	207	450	550

7.25 Figure 7.33 shows the tensile engineering stress–strain behavior for a steel alloy.

(a) What is the modulus of elasticity?

(b) What is the proportional limit?

(c) What is the yield strength at a strain offset of 0.002?

(d) What is the tensile strength?

7.26 A cylindrical specimen of a brass alloy having a length of 60 mm (2.36 in.) must elongate only 10.8 mm (0.425 in.) when a tensile load of 50,000 N (11,240 lb_f) is applied. Under these circumstances, what must be the radius of the specimen? Consider this brass alloy to have the stress–strain behavior shown in Figure 7.12.

7.27 A load of 44,500 N (10,000 lb_f) is applied to a cylindrical specimen of steel (displaying the stress–strain behavior shown in Figure 7.33) that has a cross-sectional diameter of 10 mm (0.40 in.).

(a) Will the specimen experience elastic or plastic deformation? Why?

(b) If the original specimen length is 500 mm (20 in.), how much will it increase in length when this load is applied?

7.28 A bar of a steel alloy that exhibits the stress–strain behavior shown in Figure 7.33 is subjected to a tensile load; the specimen is 300 mm (12 in.) long, and of square cross section 4.5 mm (0.175 in.) on a side.

(a) Compute the magnitude of the load necessary to produce an elongation of 0.46 mm (0.018 in.).

(b) What will be the deformation after the load is released?

7.29 A cylindrical specimen of aluminum having a diameter of 0.505 in. (12.8 mm) and a gauge length of 2.000 in. (50.800 mm) is pulled in tension. Use the load–elongation characteristics tabulated below to complete problems a through f.

Load		*Length*	
lb_f	*N*	*in.*	*mm*
0	0	2.000	50.800
1,650	7,330	2.002	50.851
3,400	15,100	2.004	50.902
5,200	23,100	2.006	50.952
6,850	30,400	2.008	51.003
7,750	34,400	2.010	51.054
8,650	38,400	2.020	51.308
9,300	41,300	2.040	51.816
10,100	44,800	2.080	52.832
10,400	46,200	2.120	53.848
10,650	47,300	2.160	54.864
10,700	47,500	2.200	55.880
10,400	46,100	2.240	56.896
10,100	44,800	2.270	57.658
9,600	42,600	2.300	58.420
8,200	36,400	2.330	59.182
	Fracture		

(a) Plot the data as engineering stress versus engineering strain.

(b) Compute the modulus of elasticity.

(c) Determine the yield strength at a strain offset of 0.002.

(d) Determine the tensile strength of this alloy.

(e) What is the approximate ductility, in percent elongation?

(f) Compute the modulus of resilience.

7.30 A specimen of ductile cast iron having a rectangular cross section of dimensions 4.8 mm × 15.9 mm ($\frac{3}{16}$ in. × $\frac{5}{8}$ in.) is deformed in tension. Using the load-elongation data tabulated below, complete problems a through f.

Load		*Length*	
N	*lb_f*	*mm*	*in.*
0	0	75.000	2.953
4,740	1065	75.025	2.954
9,140	2055	75.050	2.955
12,920	2900	75.075	2.956
16,540	3720	75.113	2.957
18,300	4110	75.150	2.959
20,170	4530	75.225	2.962
22,900	5145	75.375	2.968
25,070	5635	75.525	2.973
26,800	6025	75.750	2.982
28,640	6440	76.500	3.012
30,240	6800	78.000	3.071
31,100	7000	79.500	3.130
31,280	7030	81.000	3.189
30,820	6930	82.500	3.248
29,180	6560	84.000	3.307
27,190	6110	85.500	3.366
24,140	5430	87.000	3.425
18,970	4265	88.725	3.493
	Fracture		

(a) Plot the data as engineering stress versus engineering strain.

(b) Compute the modulus of elasticity.

(c) Determine the yield strength at a strain offset of 0.002.

(d) Determine the tensile strength of this alloy.

(e) Compute the modulus of resilience.

(f) What is the ductility, in percent elongation?

7.31 A cylindrical metal specimen having an original diameter of 12.8 mm (0.505 in.) and gauge length of 50.80 mm (2.000 in.) is pulled in tension until fracture occurs. The diameter at the point of fracture is 6.60 mm (0.260 in.), and the fractured gauge length is 72.14 mm (2.840 in.). Calculate the ductility in terms of percent reduction in area and percent elongation.

7.32 Calculate the moduli of resilience for the materials having the stress–strain behaviors shown in Figures 7.12 and 7.33.

7.33 Determine the modulus of resilience for each of the following alloys:

Material	Yield Strength MPa	Yield Strength psi
Steel alloy	550	80,000
Brass alloy	350	50,750
Aluminum alloy	250	36,250
Titanium alloy	800	116,000

Use modulus of elasticity values in Table 7.1.

7.34 A brass alloy to be used for a spring application must have a modulus of resilience of at least 0.75 MPa (110 psi). What must be its minimum yield strength?

7.35 **(a)** Make a schematic plot showing the tensile true stress–strain behavior for a typical metal alloy.

(b) Superimpose on this plot a schematic curve for the compressive true stress–strain behavior for the same alloy. Explain any difference between this curve and the one in part a.

(c) Now superimpose a schematic curve for the compressive engineering stress–strain behavior for this same alloy, and explain any difference between this curve and the one in part b.

7.36 Show that Equations 7.18a and 7.18b are valid when there is no volume change during deformation.

7.37 Demonstrate that Equation 7.16, the expression defining true strain, may also be represented by

$$\epsilon_T = \ln\left(\frac{A_0}{A_i}\right)$$

when specimen volume remains constant during deformation. Which of these two expressions is more valid during necking? Why?

7.38 Using the data in Problem 7.29 and Equations 7.15, 7.16, and 7.18a, generate a true stress–true strain plot for aluminum. Equation 7.18a becomes invalid past the point at which necking begins; therefore, measured diameters are given below for the last four data points, which should be used in true stress computations.

Load lb_f	Load N	Length in.	Length mm	Diameter in.	Diameter mm
10,400	46,100	2.240	56.896	0.461	11.71
10,100	44,800	2.270	57.658	0.431	10.95
9,600	42,600	2.300	58.420	0.418	10.62
8,200	36,400	2.330	59.182	0.370	9.40

7.39 A tensile test is performed on a metal specimen, and it is found that a true plastic strain of 0.20 is produced when a true stress of 575 MPa (83,500 psi) is applied; for the same metal, the value of K in Equation 7.19 is 860 MPa (125,000 psi). Calculate the true strain that results from the application of a true stress of 600 MPa (87,000 psi).

7.40 For some metal alloy, a true stress of 415 MPa (60,175 psi) produces a plastic true strain of 0.475. How much will a specimen of this material elongate when a true stress of 325 MPa (46,125 psi) is applied if the original length is 300 mm (11.8 in.)? Assume a value of 0.25 for the strain-hardening exponent n.

7.41 The following true stresses produce the corresponding true plastic strains for a brass alloy:

True Stress (psi)	True Strain
50,000	0.10
60,000	0.20

What true stress is necessary to produce a true plastic strain of 0.25?

7.42 For a brass alloy, the following engineering stresses produce the corresponding plastic engineering strains, prior to necking:

Engineering Stress (MPa)	Engineering Strain
235	0.194
250	0.296

On the basis of this information, compute the *engineering* stress necessary to produce an *engineering* strain of 0.25.

7.43 Find the toughness (or energy to cause fracture) for a metal that experiences both elastic and plastic deformation. Assume Equation 7.5 for elastic deformation, that the modulus of elasticity is 172 GPa (25×10^6 psi), and that elastic deformation terminates at a strain of 0.01. For plastic deformation, assume that the relationship between stress and strain is described by Equation 7.19, in which the values for K and n are 6900 MPa (1×10^6 psi) and 0.30, respectively. Furthermore, plastic deformation occurs between strain values of 0.01 and 0.75, at which point fracture occurs.

7.44 For a tensile test, it can be demonstrated that necking begins when

$$\frac{d\sigma_T}{d\epsilon_T} = \sigma_T \tag{7.31}$$

Using Equation 7.19, determine the value of the true strain at this onset of necking.

7.45 Taking the logarithm of both sides of Equation 7.19 yields

$$\log \sigma_T = \log K + n \log \epsilon_T \tag{7.32}$$

Thus, a plot of log σ_T versus log ϵ_T in the plastic region to the point of necking should yield a straight line having a slope of n and an intercept (at log $\sigma_T = 0$) of log K.

Using the appropriate data tabulated in Problem 7.29, make a plot of log σ_T versus log ϵ_T and determine the values of n and K. It will be necessary to convert engineering stresses and strains to true stresses and strains using Equations 7.18a and 7.18b.

7.46 A cylindrical specimen of a brass alloy 7.5 mm (0.30 in.) in diameter and 90.0 mm (3.54 in.) long is pulled in tension with a force of 6000 N (1350 lb_f); the force is subsequently released.

(a) Compute the final length of the specimen at this time. The tensile stress–strain behavior for this alloy is shown in Figure 7.12.

(b) Compute the final specimen length when the load is increased to 16,500 N (3700 lb_f) and then released.

7.47 A steel specimen having a rectangular cross section of dimensions 19 mm × 3.2 mm ($\frac{3}{4}$ in. × $\frac{1}{8}$ in.) has the stress–strain behavior shown in Figure 7.33. If this specimen is subjected to a tensile force of 33,400 N (7,500 lb_f), then

(a) Determine the elastic and plastic strain values.

(b) If its original length is 460 mm (18 in.), what will be its final length after the load in part a is applied and then released?

7.48 A three-point bending test is performed on a glass specimen having a rectangular cross section of height d 5 mm (0.2 in.) and width b 10 mm (0.4 in.); the distance between support points is 45 mm (1.75 in.).

(a) Compute the flexural strength if the load at fracture is 290 N (65 lb_f).

(b) The point of maximum deflection Δy occurs at the center of the specimen and is described by

$$\Delta y = \frac{FL^3}{48EI}$$

where E is the modulus of elasticity and I the cross-sectional moment of inertia. Compute Δy at a load of 266 N (60 lb_f).

7.49 A circular specimen of MgO is loaded using a three-point bending mode. Compute the minimum possible radius of the specimen without fracture, given that the applied load is 425 N (95.5 lb_f), the flexural strength is 105 MPa (15,000 psi), and the separation between load points is 50 mm (2.0 in.).

7.50 A three-point bending test was performed on an aluminum oxide specimen having a circular cross section of radius 3.5 mm (0.14 in.); the specimen fractured at a load of 950 N (215 lb_f) when the distance between the support points was 50 mm (2.0 in.). Another test is to be performed on a specimen of this same material, but one that has a square cross section of 12 mm (0.47 in.) length on each edge. At what load would you expect this specimen to fracture if the support point separation is 40 mm (1.6 in.)?

7.51 **(a)** A three-point transverse bending test is conducted on a cylindrical specimen of aluminum oxide having a reported flexural strength of 390 MPa (56,600 psi). If the specimen radius is 2.5 mm (0.10 in.) and the support point separation distance is 30 mm (1.2 in.), predict whether or not you would expect the specimen to fracture when a load of 620 N (140 lb_f) is applied. Justify your prediction.

(b) Would you be 100% certain of the prediction in part a? Why or why not?

7.52* The modulus of elasticity for beryllium oxide (BeO) having 5 vol% porosity is 310 GPa (45×10^6 psi).

(a) Compute the modulus of elasticity for the nonporous material.

(b) Compute the modulus of elasticity for 10 vol% porosity.

7.53* The modulus of elasticity for boron carbide (B_4C) having 5 vol% porosity is 290 GPa (42×10^6 psi).

(a) Compute the modulus of elasticity for the nonporous material.

(b) At what volume percent porosity will the modulus of elasticity be 235 GPa (34×10^6 psi)?

7.54* Using the data in Table 7.2, do the following:

(a) Determine the flexural strength for nonporous MgO assuming a value of 3.75 for n in Equation 7.22.

(b) Compute the volume fraction porosity at which the flexural strength for MgO is 62 MPa (9000 psi).

7.55* The flexural strength and associated volume fraction porosity for two specimens of the same ceramic material are as follows:

σ_{fs} (MPa)	P
100	0.05
50	0.20

(a) Compute the flexural strength for a completely nonporous specimen of this material.

(b) Compute the flexural strength for a 0.1 volume fraction porosity.

7.56 From the stress–strain data for polymethyl methacrylate shown in Figure 7.24, determine the modulus of elasticity and tensile strength at room temperature [20°C (68°F)], and compare these values with those given in Tables 7.1 and 7.2.

7.57 When citing the ductility as percent elongation for semicrystalline polymers, it is not necessary to specify the specimen gauge length, as is the case with metals. Why is this so?

7.58* In your own words, briefly describe the phenomenon of viscoelasticity.

7.59* For some viscoelastic polymers that are subjected to stress relaxation tests, the stress decays with time according to

$$\sigma(t) = \sigma(0) \exp\left(-\frac{t}{\tau}\right) \qquad (7.33)$$

where $\sigma(t)$ and $\sigma(0)$ represent the time-dependent and initial (i.e., time = 0) stresses, respectively, and t and τ denote elapsed time and the relaxation time; τ is a time-independent constant characteristic of the material. A specimen of some viscoelastic polymer the stress relaxation of which obeys Equation 7.33 was suddenly pulled in tension to a measured strain of 0.6; the stress necessary to maintain this constant strain was measured as a function of time. Determine $E_r(10)$ for this material if the initial stress level was 2.76 MPa (400 psi), which dropped to 1.72 MPa (250 psi) after 60 s.

7.60* In Figure 7.35, the logarithm of $E_r(t)$ versus the logarithm of time is plotted for polyisobutylene at a variety of temperatures. Make a plot of log $E_r(10)$ versus temperature and then estimate the T_g.

7.61* On the basis of the curves in Figure 7.26, sketch schematic strain-time plots for the following polystyrene materials at the specified temperatures:

(a) Amorphous at 120°C.

(b) Crosslinked at 150°C.

(c) Crystalline at 230°C.

(d) Crosslinked at 50°C.

7.62* **(a)** Contrast the manner in which stress relaxation and viscoelastic creep tests are conducted.

(b) For each of these tests, cite the experimental parameter of interest and how it is determined.

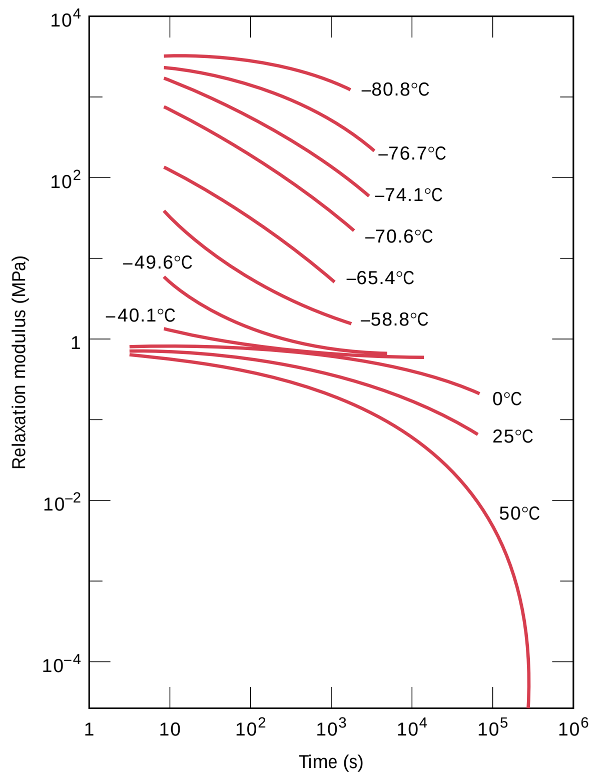

FIGURE 7.35 Logarithm of relaxation modulus versus logarithm of time for polyisobutylene between −80 and 50°C. (Adapted from E. Catsiff and A. V. Tobolsky, ''Stress-Relaxation of Polyisobutylene in the Transition Region [1,2],'' *J. Colloid Sci.*, **10,** 377 [1955]. Reprinted by permission of Academic Press, Inc.)

7.63* Make two schematic plots of the logarithm of relaxation modulus versus temperature for an amorphous polymer (curve *C* in Figure 7.29).

(a) On one of these plots demonstrate how the behavior changes with increasing molecular weight.

(b) On the other plot, indicate the change in behavior with increasing crosslinking.

7.64 **(a)** A 10-mm-diameter Brinell hardness indenter produced an indentation 1.62 mm in diameter in a steel alloy when a load of 500 kg was used. Compute the HB of this material.

(b) What will be the diameter of an indentation to yield a hardness of 450 HB when a 500 kg load is used?

7.65 Estimate the Brinell and Rockwell hardnesses for the following:

(a) The naval brass for which the stress–strain behavior is shown in Figure 7.12.

(b) The steel for which the stress–strain behavior is shown in Figure 7.33.

7.66 Using the data represented in Figure 7.31, specify equations relating tensile strength and Brinell hardness for brass and nodular cast iron, similar to Equations 7.25a and 7.25b for steels.

7.67 Cite five factors that lead to scatter in measured material properties.

7.68* Below are tabulated a number of Rockwell B hardness values that were measured on a single steel specimen. Compute average and standard deviation hardness values.

83.3	80.7	86.4
88.3	84.7	85.2
82.8	87.8	86.9
86.2	83.5	84.4
87.2	85.5	86.3

7.69 Upon what three criteria are factors of safety based?

7.70 Determine working stresses for the two alloys the stress–strain behaviors of which are shown in Figures 7.12 and 7.33.

Design Problems

7.D1 A large tower is to be supported by a series of steel wires. It is estimated that the load on each wire will be 11,100 N (2500 lb_f). Determine the minimum required wire diameter assuming a factor of safety of 2 and a yield strength of 1030 MPa (150,000 psi).

7.D2 **(a)** Gaseous hydrogen at a constant pressure of 1.013 MPa (10 atm) is to flow within the inside of a thin-walled cylindrical tube of nickel that has a radius of 0.1 m. The temperature of the tube is to be 300°C and the pressure of hydrogen outside of the tube will be maintained at 0.01013 MPa (0.1 atm). Calculate the minimum wall thickness if the diffusion flux is to be no greater than 1×10^{-7} mol/m^2-s. The concentration of hydrogen in the nickel, C_H (in moles hydrogen per m^3 of Ni) is a function of hydrogen pressure, p_{H_2} (in MPa) and absolute temperature (T) according to

$$C_H = 30.8 \sqrt{p_{H_2}} \exp\left(-\frac{12.3 \text{ kJ/mol}}{RT}\right) \tag{7.34}$$

Furthermore, the diffusion coefficient for the diffusion of H in Ni depends on temperature as

$$D_H(\text{m}^2/\text{s}) = 4.76 \times 10^{-7} \exp\left(-\frac{39.56 \text{ kJ/mol}}{RT}\right) \tag{7.35}$$

(b) For thin-walled cylindrical tubes that are pressurized, the circumferential stress is a function of the pressure difference across the wall (Δp), cylinder radius (r), and tube thickness (Δx) as

$$\sigma = \frac{r\,\Delta p}{4\,\Delta x} \tag{7.36}$$

Compute the circumferential stress to which the walls of this pressurized cylinder are exposed.

(c) The room-temperature yield strength of Ni is 100 MPa (15,000 psi) and, furthermore, σ_y diminishes about 5 MPa for every 50°C rise in temperature. Would you expect the wall thickness computed in part (b) to be suitable for this Ni cylinder at 300°C? Why or why not?

(d) If this thickness is found to be suitable, compute the minimum thickness that could be used without any deformation of the tube walls. How much would the diffusion flux increase with this reduction in thickness? On the other hand, if the thickness determined in part (c) is found to be unsuitable, then specify a minimum thickness that you would use. In this case, how much of a diminishment in diffusion flux would result?

7.D3 Consider the steady-state diffusion of hydrogen through the walls of a cylindrical nickel tube as described in Problem 7.D2. One design calls for a diffusion flux of 5×10^{-8} mol/m^2-s, a tube radius of 0.125 m, and inside and outside pressures of 2.026 MPa (20 atm) and 0.0203 MPa (0.2 atm), respectively; the maximum allowable temperature is 450°C. Specify a suitable temperature and wall thickness to give this diffusion flux and yet ensure that the tube walls will not experience any permanent deformation.

7.D4 It is necessary to select a ceramic material to be stressed using a three-point loading scheme (Figure 7.18). The specimen must have a circular cross section and a radius of 2.5 mm (0.10 in.), and must not experience fracture or a deflection of more than 6.2×10^{-2} mm (2.4×10^{-3} in.) at its center when a load of 275 N (62 lb_f) is applied. If the distance between support points is 45 mm (1.77 in.), which of the ceramic materials in Tables 7.1 and 7.2 are candidates? The magnitude of the centerpoint deflection may be computed using the equation supplied in Problem 7.48.